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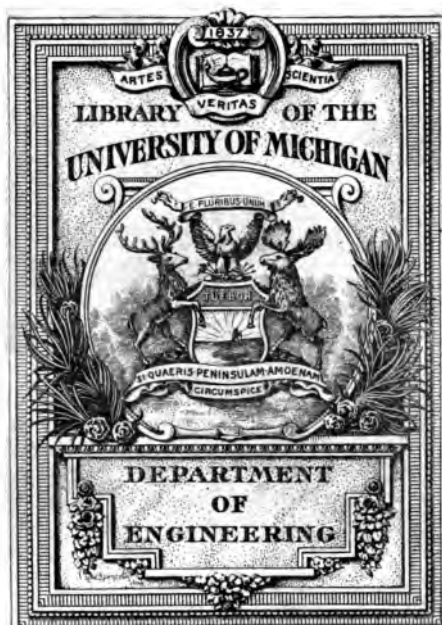

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PREFACE TO SECOND EDITION.

THE earlier edition of this series of essays having been exhausted, a new issue has been determined upon.

The methods of surveying herein described accords so completely with the latest practice, that this new edition seems quite certain to prove even more widely useful than the former one.

The text and diagrams have been revised.

GEO. W. PLYMPTON.

PREFACE.

THE essays republished in this little volume have already won the approval of practical surveyors, having appeared as original contributions in the ENGINEERING MAGAZINE.

It is in answer to an increasing demand for a god guide to modern methods of surveying areas that the articles have been brought together to form a single volume.

ON

TOPOGRAPHICAL SURVEYING.

BY

GEO. J. SPECHT, C. E.

ON TOPOGRAPHICAL SURVEYING.

THE object of Topography is to determine the relative positions of points of the earth's surface, that can be referred without error to a tangent plane, and therefore independent of the sphericity of the globe.

The operations of a topographical survey, consequently, are two—namely, to first project a system of points upon such a tangent plane; and, secondly, to find the distances of the same above or below the plane; or, in other words, to measure the lengths of the projecting normals. The first process is ordinary surveying; the second, leveling.

The results of a topographical survey are laid down in a so-called topographical map, which is a representation or complete image of the ground on a reduced scale.

Topographical maps are of the greatest convenience in locating railroads or other roads, in planning irrigation works, draining works, in mining enterprises, in military operations, &c., &c. In a topographical map the configuration of the ground is reduced to an image, which represents to the eye a large area at one glance, which in nature could not be viewed but by many separate inspections; therefore, the judgment about the relation of the different parts of the work will be a clearer and more intelligent one. This refers especially to mining work, where very frequently the problem occurs, to strike a vein with a tunnel in a certain level. In this problem a correct topographical map will often save the mining company several hundred feet of tunnel work, or, in other words, thousands of dollars.

One reason why topographical surveys are not oftener made, is certainly the slowness on one hand and the inaccuracy on the other hand of the old methods.

Two different methods have heretofore been employed; one has the great disadvantage of slowness, and the other that of being unreliable. The first is a combination of common surveying with leveling. Provided these two operations are carried out with all possible care, the work will be a very exact one; which, however, will partly be lost by the inaccuracy of the drawing. Therefore, in this method, the field-work is unnecessarily superior to the requirements of the case, as the reduced scale of a topographical map does not allow the representation of the smaller details. And as the topographical map is the first and direct object of a topographical survey, the latter ought not to be more exact than the scale of the map requires; for instance, if the map is made on a scale of 1-5000 ($1'' = 416.6'$) the distances on the map can be read or estimated with any certainty only within four feet. Consequently, the survey does not need to be more detailed than to correspond to this limit. The second method is also

a combination of common surveying with leveling, yet in a more hurried and therefore unreliable manner; it is on ground of a measured and leveled base to sketch the surroundings. As a matter of course the correctness of a topographical map, derived from such a survey, depends entirely on the ability of tho topographer to estimate the relations between the different points. And as there are too many sources of error such topographical maps are but little value; they render good service in military reconnoissance, but hardly anywhere else.

Without going into the details of the old methods (which are shortly treated in Gillespie's Handbook) I shall proceed at once to give an account of the new method of topographical surveying. The word "new" is justified only in view of the two above-mentioned methods, as the one to be described has been known since 1852, when the Italian Professor and Officer of Artillery, Porro, of Milan, gave an account of this method

Annales des Ponts et Chaussees, and when also the French engineer, Minot, published a treatise on this subject. The French and the Italian were the first who used it; then it came largely into use in Switzerland, where, in connection with the plane table, it was and still is used for the beautiful and masterly-executed topographical maps of Switzerland, in a scale of 1.50,000, with contour lines of 30 meters distance. Austria and Germany followed next, and are using it largely in railroads and similar enterprises. To-day it is used in those countries wherever any work of that kind is done; the Prussian General Staff employs it nearly exclusively. When and to what extent it was introduced in the United States is not known to the writer, but the note of A. S. Hardy, Professor of Civil Engineering in the Chandler Scientific School, in *Van Nostrand's Engineering Magazine*, Aug., 1877, indicates that this method was then hardly known, for otherwise he would not have praised the old, old method

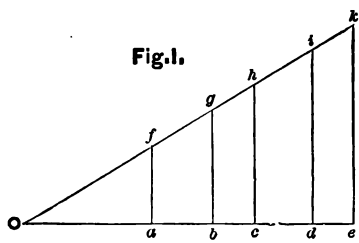
he mentions, as quite a new application of contour lines. The United States Coast Survey uses this new method extensively in connection with the plane table.

THE NEW METHOD

of topographical surveying consists in simultaneously obtaining the horizontal and vertical positions of a point; in other words, each point is determined by *one* operation in reference to its horizontal and vertical location. This is accomplished by the use of a transit with the so-called stadia wires and a vertical circle, and a leveling rod or so-called telemeter or stadia-rod.

Besides the ordinary horizontal and vertical cross hairs of the diaphragm of the telescope, two extra horizontal hairs are placed parallel with the centre one and equally distant on each side of it, which, if the telescope is sighted at a leveling rod, will inclose a part of this rod or stadia rod, proportional to the distance from the instrument to the rod. By this arrangment we have obtained

an angle of sight, which remains always constant. Supposing the eye to be in the point O (Fig. 1), the lines O*e* and



O*k* represent the lines of sight from the eye through the stadia-wires to the rod, which stands consecutively at *ke*, *id*, *hc*, *gb* and *fa*. According to a simple geometrical theorem we have the following proportion:

O*a* : O*b* : O*c* : O*d* : O*e* : = *af* : *bg* : *ch* : *di* : *ek*, which means that the reading of the rod placed on the different points *a*, *b*, *c*, *d* and *e* is proportional to the distances O*a*, O*b*, O*c*, O*d* and O*e*.

The system of lenses which constitute the telescope do not allow the use of

this proportion directly in stadia measurements, because distances must be counted from a point in front of the object glass at a distance equal to the focal length of that lens.

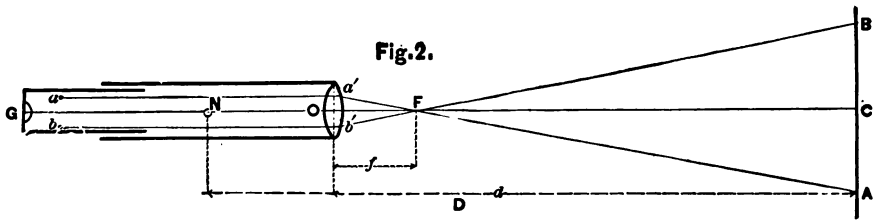
Fig. 2 represents the section of a common telescope with but two lenses, between which the diaphragm with the stadia-wires is placed.

We assume:

f = the local distance of the object glass.
p = the distance of the stadia-wires a and b from each other.
d = the horizontal distance of the object glass to the stadia.
a = stadia reading (B A).
D = horizontal distance from middle of instrument to stadia.

The telescope is leveled and sighted to a leveling or stadia rod, which is held vertically, hence at a right angle with the line of sight. According to a principle of optics, rays parallel to the axis of the lens meet after being refracted in the focus of the lens. Suppose the two stadia wires are the sources of those

Fig.2.



rays, we have, from the similarity of the two triangles, a' b' F and F A B the proportion:

$$(d—f) : a = f : p.$$

The value of the quotient, $f : p$, is, or at least can be made, a constant one, which we will designate by the letter k; hence we have:

$$(d—f) = \mathrm{FC} = ka.$$

In order to get the distance from the center of the instrument N, we have to add to the above value of FC yet the value c.

$$c = \mathrm{OF} + \mathrm{ON}.$$

ON is mostly equal to half the focal length of the object glass, hence we have

$$\mathrm{C} = f + f_2 = 1.5f.$$

Therefore the formula for the distance of the stadia from the center of instrument, when that stadia is at right angles to the level line of sight, is:

(1) $$\mathrm{D} = ka + c.$$

When the line of sight is not level, but the stadia at right angles to it, the formula for the horizontal distance is:

(2) $$D = ka\cos n + c + om.$$

The member $\overline{om} = \frac{a}{2}\sin n$; for $a = 24'$,

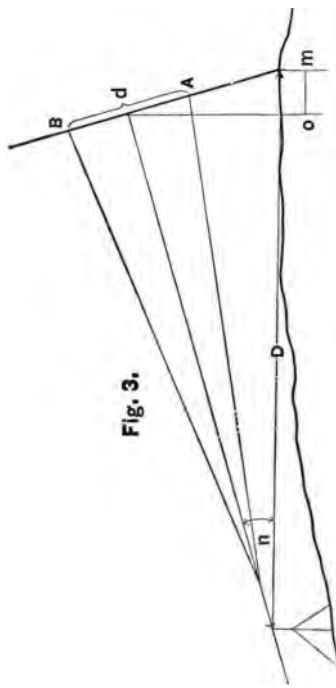


Fig. 3.

$n = 45°$ the value of $\overline{om}$ is but 8.4′, and for $a = 10'$, $n = 10°$ it is 0.86′; this shows that $\overline{om}$ in most cases may safely be omitted.

Some engineers let the rodman hold the staff perpendicular to the line of sight; they accomplish this by different devices, as, a telescope or a pair of sights attached at right angle to the staff. This method is not practicable, as it is very difficult, especially in long distances, and with vertical angles for the rodman to see the exact position of the telescopes, and furthermore, in some instances it is entirely impossible, when, for instance, the point to be ascertained is on a place where only the staff can stand, but where there is no room for the man. The only correct way to hold the staff is vertically.

In this case we have the following: (Fig. 4).

$MF = c + GF = c + k.C.D.$

CD must be expressed by AB.

$AB = a.$ $AGB = 2m.$

$CD = 2GF \tan. m.$

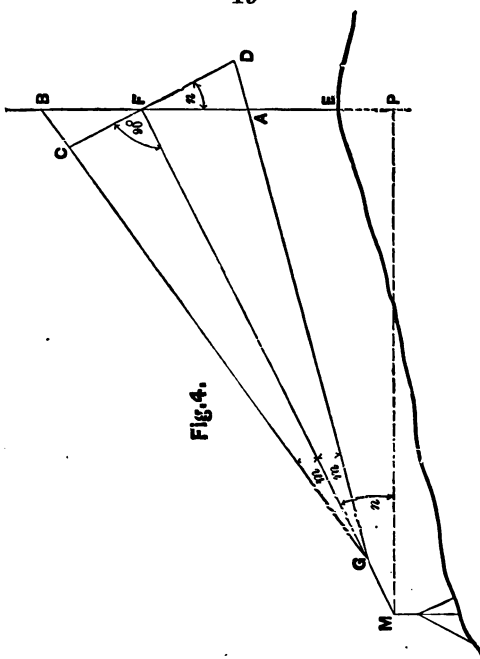


Fig. 4.

By the similarity of the two triangles AGF and BGF, we have after some transformations

$$AF + BF = GF \sin m \left(\frac{1}{\cos (n + m)} + \frac{1}{\cos (n - m)} \right)$$

$$GF = \frac{CD}{2 \tan m}, \quad AF + BF = a$$

$$a = CD.$$

$$\frac{\cos m . \sin m \cos (n - m) + \cos (n + m)}{2 \sin m \cos (n + m) \cos (n - m)}$$

$$CD = a \frac{\cos^2 n \cos^2 m - \sin^2 n \sin^2 m}{\cos n \cos m}$$

$$MF = c + GF = c + k.CD. MF = \frac{D'}{\cos n}$$

$$\frac{D}{\cos n} = c + k.a \frac{(\cos^2 n \cos^2 m - \sin^2 n \sin^2 m)}{\cos n \cos^2 m}$$

$$D = c. \cos n + a.k. \cos^2 n - a.k. \sin^2 n \tan^2 m.$$

The last term may safely be neglected, as it is very small, even for long distances and large angles of elevation (for 1500′, $n = 45°$ and $k = 100$, it is but 0.07′). Therefore, the final formula for distances, with a stadia kept vertical, and with wires equidistant from the centre wire is the following:

(3) $$D = c. \cos n + a.k. \cos^2 n.$$

The value of $c. \cos n$ is usually neglected, as it amounts to but 1 or 1.5 feet; it is exact enough to add always 1.25′ to the

distance as derived from the formula without considering the different values of the angle n.

$$(3a) \qquad D = a.k.\cos^2 n$$

In order to make the subtraction of the readings of the upper and lower wire quickly, place one of the latter on the division of a whole foot, and count the parts included between this and the other wire: this multiply mentally by 100 (the constant k) which gives the direct distance d'.

In cases where it is not possible to read with both stadia wires, it is the custom to use but one of them in connection with the center wire, and then to double the reading thus obtained. With very large vertical angles, this custom is not advisable, as is shown by the following theoretical investigation:

Take the same figure 4, as above,

$$BF = \frac{BG \sin m}{\sin.(90° + n)}; \quad AF = \frac{AG \sin m}{\sin (90° - n)}$$

$$BF = \frac{GF.\sin m}{\sin (90° - m - u)};$$

$$AF = \frac{GF.\sin m}{\sin(90 - m + n)}$$

$$\frac{BG \sin m}{\sin(90° + n)} = \frac{GF.\sin m}{\sin[90° - (n + m)]}$$

$$\frac{BG}{\cos n} = \frac{GF}{\cos(n + m)}; \quad BG = \frac{GF \cos n}{\cos(n + m)};$$

$$GA = \frac{GF \cos n}{\cos(n = m)}$$

$$BG : AG = \frac{GF \cos n}{\cos(n + m)} : \frac{GF \cos n}{\cos(n - m)}$$

$$BG : AG = \cos(n - m) : \cos(n + m)$$

$$BF : (BF + AF) = \cos(n - m) :$$
$$[\cos(n - m) + \cos(n + m)]$$

$$AF : (BF + AF) = (\cos n + m) :$$
$$[\cos(n + m) + \cos(n - m)]$$

$$\frac{BF.2\cos n \cos m}{\cos n \cos m + \sin n \sin m}$$
$$= \frac{AF.2\cos n \cos m}{\cos n \cos m - \sin n \sin m}$$

$$\frac{BF \cos n \cos m}{\cos n \cos m} - \frac{BF \sin n \sin m}{\cos n \cos m}$$
$$= \frac{AF \cos n \cos m}{\cos n \cos m} + \frac{AH \sin n \sin m}{\cos n \cos m}$$

$$BF(1 - \tan n \tan m) = AF(1 + \tan n \tan m)$$

$$BF = \frac{a}{2}\,;\ AF = \frac{a}{2}$$

Now, if we multiply one of these values by 2, we see that the result is not equal to a, but equal to $a \pm a \tan n \tan m$; hence the distance D is either too long or too short by the amount of $a.\ k.\ \cos^2 \tan n \tan m$; for $a = 15'$, $n = 45°$ and $k = 100$, the distance measured with both stadia wires is 749.7′, but as measured with only one stadia wire and the centre wire, we have either 753.4′ or 746.0′, which is an error of 0.50%; for $a = 15'$, $n = 22°$ and $k = 100$, we have correct distance = 1289.1′, distance with one wire, either 1286.5′ or 1291.7′, which is an error of 0.25%.

To find the height of the point where the stadia stands, simultaneously with the distance, we have the following:

We assume, in reference to figure 4,

q = height of instrument point above datum.

MP = D = horizontal distance as derived from formula (3).

n = vertical angle.

h = FE = stadia reading of the centre wire.

Q = height of stadia point above datum; it is,

$$Q = q + D\tan n - h.$$

The subtraction of h can be made directly by the instrument by sighting with the centre wire to that point of the rod, which is equal to the height of the telescope above the ground (which is in most cases = 4.5′); g will be constant for one and the same instrument point; then the above formula:

$$Q = D\tan n;$$

this, in connection with formula (3) gives

$$Q = c\sin n + a.k.\cos n.\sin n.$$

$$Q = c\sin n + a.k.\frac{\sin 2n}{2}.$$

The first form of the equation can be neglected when the vertical angle is not too large; hence the final formula for the height is

$$(5) \qquad Q = \frac{a.k.\sin 2n}{2}.$$

The position of the stadia must be strictly vertical.

Without giving here the theoretical investigation of the manner in which an inclination of the stadia towards or from the instrument affects the distance, I shall mention but the results of the investigation on this subject. The following table is calculated from a formula given by Professor Helmert, of the Royal Polytechnic School in Aachen (Germany):

$n.$	$a = 2.0$	3.0	4.0	5.0	6.0	8.0	10.0	12.0
10°	0.34	0.43	0.54	0.62	0.76	0.99	1.22	1.46
	1.10	1.12	1.18	1.24	1.30	1.42	1.62	1.89
	2.42	2.43	2.46	2.48	2.51	2.59	2.70	2.80
20°	0.64	0.32	1.01	1.18	1.43	1.88	2.32	2.76
	2.08	2.16	2.23	2.33	2.25	2.72	3.06	3.56
	4.57	4.60	4.65	4.68	4.79	4.82	5.10	5.28
30°	0.87	1.11	1.37	1.59	1.93	2.53	3.13	3.72
	2.82	2.92	3.01	3.14	3.31	3.68	4.13	4.80
	6.18	6.20	6.26	6.31	6.38	6.60	6.85	7.15
40°	0.98	1.25	1.55	1.81	2.20	2.88	3.56	4.22
	3.19	3.30	3.41	3.58	3.77	4.18	4.68	5.45
	7.00	7.05	7.11	7.18	7.25	7.49	7.80	8.12

$$D - D' = \pm \tfrac{1}{2} k \sin 2n \sin o \sqrt{2m^2 + \frac{a^2}{2}}$$

where D is the reading at a stadia exactly vertical.

D′ is the reading at a stadia not vertical.

k = the constant, n = vertical angle, o = angle of inclination of the stadia when not held exactly vertical, m = height of the center wire, and a = stadia reading.

The table is calculated for $k = 100$, sin $o = 0.01$, and for $m = 1'$, $4.5'$ and $10'$.

The error increases with the height of m; in shorter distances the result is sevenfold better when the center wire is placed as low as one foot than it is at $10'$; in longer distances this advantage is only double.

It is always better to place the center wire as low as possible. If the stadia is provided with a good circular level, the rodman ought to be able to hold it vertical within 500 seconds; that means, that the inclination of the stadia shall not be more than $0.023'$ in a $10'$ stadia, or $0.034'$ in a stadia of $15'$ length.

DETERMINATION OF THE TWO CONSTANT CO-EFFICIENTS t AND k.

Although the stadia wires are usually arranged so that the reading of one foot signifies a distance of 100 feet, I will explain here how to determine the value of it for any case. Suppose the engineer goes to work without knowing his constant, and not having adjustable stadia wires. The operation then is as follows:

Measure off on a level ground a straight line of about 1000′ length; mark every 100′, place the instrument above the starting point, and let the rodman place his rod on each of the points measured off; note the reading of all three wires separately, repeat this operation four times; the telescope must be as level as the ground allows; measure the exact height of the instrument, *i. e.*, the height of the telescope axis above the ground. Then find the difference between upper (o) and middle (m) wire; between middle (m) and lower (n) wire, and between upper (o) and lower (n) wire, from the

four different values for each difference determine the average value; then solve the equation for the horizontal distance (1) $D = k.a. + c.$, with the different average values, and you find the value of k and c. In case the stadia wires should not be equidistant from the center wire there will be three different constants, one for the use of the upper and middle, one for the use of the middle and lower, and one for the upper and lower wire. The following example, which occurred to me, will explain these rules (the measures are meters, which, of course, make no difference):

No. of Observations.	Distance Measured.	Stadia Reading.			Differences.			Angle of Elevation.	Height of instrument.
		o.	*m.*	*u.*	*o*—*m.*	*m*—*u.*	*o*—*u.*		
	Meters.								
	25	1.503	1.300	1.117	0.203	0.183	0.386	0° 32′	
	50	1.503	1.100	0.727	0.403	0.373	0.776		
1.	75	1.810	1.200	0.640	0.610	0.560	1.170	1° 0′	
	100	1.915	1.100	0.355	0.815	0.745	1.560		
	115	1.940	1.000	0.140	0.940	0.860	1.800	1° 2′	
	115	1.940	1.000	0.140	0.940	0.860	1.800		1.318
	100	1.915	1.100	0.350	0.815	0.750	1.565		
2.	75	1.810	1.200	0.640	0.610	0.560	1.170		
	50	1.507	1.100	0.730	0.407	0.370	0.777		
	25	1.700	1.500	1.316	0.200	0.184	0.384		
	25	1.502	1.300	1.116	0.202	0.184	0.386		
	50	1.504	1.100	0.730	0.404	0.370	0.774		
3.	75	1.812	1.200	6.642	0.612	0.558	1.170		
	100	1.915	1.100	0.355	0.815	0.735	1.560		
	115	1.940	1.000	0.140	0.940	0.860	1.800		
	115	1.937	1.000	0.140	0.937	0.860	1.797		1.318
	100	1.915	1.100	0.355	0.815	0.745	1 560		
4.	75	1.810	1.200	0.640	0.610	0.560	1.170		
	50	1.505	1.100	0.728	0.405	0 372	0.777		
	25	1.701	1.500	1.315	0.201	0.185	0.386		

The base was 115^{u} long.

Out of these observations we derive the following means:

Distances.	Differences.		
	$o-m$.	$m-u$.	$o-u$.
24	0.201	0.184	0.385
50	0.405	0.371	0.776
75	0.610	0.560	1.170
100	0.815	0.746	1.561
115	0.939	0.860	1.799

The different values of these differences, applied to the formula for horizontal distances (the angle of elevation is so small that it need not be considered),

$$D = k.a + c,$$

we have the following fifteen equations:

Equations for $(o-m)$

$$\text{I.} \begin{cases} 25^{m} = 0{,}201k + c. \\ 50^{m} = 0.405k + c. \\ 75^{m} = 0.610k + c. \\ 100^{m} = 0.815k + c. \\ 115^{m} = 0.939k + c. \end{cases}$$

Equations for $(m—n)$

$$\text{II.}\begin{cases} 25^m = 0.184k + c'. \\ 50^m = 0.371k + c'. \\ 75^m = 0.560k + c'. \\ 100^m = 0.746k + c'. \\ 115^m = 0.860k + c'. \end{cases}$$

Equations for $(o—n)$

$$\text{III.}\begin{cases} 25^m = 0.385k + c''. \\ 50^m = 0.776k + c''. \\ 75^m = 1.170k + c''. \\ 100^m = 1.561k + c''. \\ 115^m = 1.799k + c''. \end{cases}$$

By solving these equations, we obtain the following average values for the constants k and c, k' and c', and k'' and c''.

For the group I, we have:

$$k = 122.30,\ b = 0.40.$$

For group II.:

$$k = 133.30,\ b' = 0.45.$$

For group III.:

$$k'' = 63.70,\ b'' = 0.50.$$

This example shows one of the most unfavorable cases, as we obtain three different values for each of the two constants, because the stadia wires are not

equi-distant from the center-wire. If the stadia wires are adjustable, the engineer has it in his power to adjust them so that the constant $k = 100$, and $k_2 = 200$, which he accomplishes by actual trial along a carefully measured straight and level line.

The constant C, which is one and a half times the total length of the object-glass, can be found closely enough for this purpose by focussing the telescope for a sight of average distance, and then measuring from the outside of the object-glass to the capstan-head-screws of the cross-hairs. This constant must be added to every stadia sight; it may be neglected for longer distances.

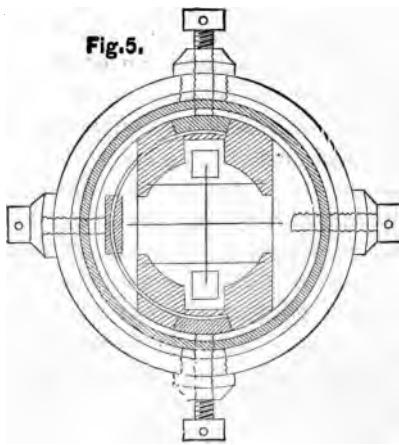
THE INSTRUMENTS

used in this method are the following:

1. A transit or theodolite, which in general construction is like the common one; the only new features of it are the stadia wires and the vertical arc.

The diaphgram carrying the cross wires has two sides, which can be moved by

Fig.5.



small capstan head screws, and on each end of which one stadia wire is fastened; an inserted spring makes their position more steady. By means of those screws the distance of the stadia wires from the center wire and from each other can be adjusted at will.

For stadia measurement it is far preferable to use a telescope with inverting eyepiece as they allow a longer distance to be read; the little inconvenience at first

of seeing the objects inverted will very soon be overcome, and the engineer will gladly adopt it, because it enlarges the range of his work so advantageously. Light and magnifying power are the essential points for a telescope used in stadia measurements, more than in any other branch of surveying. Therefore, the telescope ought to have none but the two-lens negative eye-piece, which inverts the objects. The so called Kellner's orthoscopic eye-piece should be used (Fig. 6); it is completely achromatic,

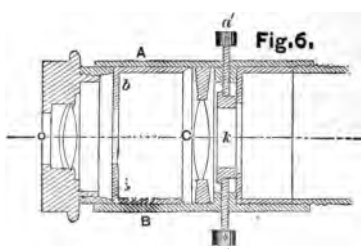


Fig.6.

and has the great advantage, which no other eye-piece has, of an actually flat field and a straight flat image of any object, correct in perspective, distinct in its

whole extent. It consists of three lenses, the bi-convex collective lens C, the flatter curve of which is towards the object-glass, and the achromatic lens O, which is composed of two lenses, similar to the achromatic object-glass. The diaphgram *b*, *b*, is a further peculiarity of this eye-piece. Messrs. Buff & Berger in Boston use such eye-pieces in their instruments.

The vertical arc must be larger than usual, so as to allow of a vernier reading of at most one minute. In order to make no mistakes in reading the vertical angle, whether it be an angle of elevation or depression, the numbering of the vertical arc must be so arranged that the zero point of the arc corresponds with the zero of the vernier, when the telescope is level, and the numbers go from 0° to 360°. By this arrangement the observer knows at once whether the angle is an angle of depression or one of elevation, without using the signs of minus or plus. The now very often preferred arrangement of making the vertical arc fixed, and the vernier movable with the

telescope is far inferior to the fixed vernier and the movable arc.

A transit or theodolite fitted out in this way is called a tachometer, which means "quick-measurer," and hence this method is "tachometric."

The next instrument essential to the topographical survey is the rod or stadia rod or telemeter; this is a self-reading leveling rod, with a graduation fit to be read at a long distance. A good rod must have the following qualities:

1. It must be light and handy for transportation.

2. The graduation must be distinct and visible at long distances; it is not to be closer than one-tenth of a foot, as otherwise the reading would become confusing for longer distances. Experience teaches that smaller subdivisions can more exactly be estimated than read by a direct division.

3. It must have a good and reliable arrangement to enable the rodman to keep it in the required position.

It is advantageous to add a target to

the rod, which is used but for the most important points, especially at new stations for the instrument.

The rod consists of two or more parts, which are either entirely separated during the transport and put together by means of screws or otherwise, when used, or they are connected with each other by hinges, or are made to slide in or along each other. I am using one which consists of three separate pieces, each 5 feet long and of a cross section, as shown in Fig. 7; the ends are pro-

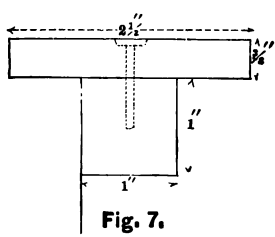


Fig. 7.

tected by iron shoes; the pieces are joined by screws. On the back is a circular level (Fig. 8).

As to the pointing of the rod, the two

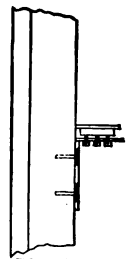


Fig. 8.

styles shown in Figs. 10 and 11 are very practical. The alternative position of the feet makes the reading a great deal easier and the whole graduation much more distinct. Fig. 11 represents a so-called "combination rod," which can

Fig. 9.

serve as a common leveling rod by means of the small subdivisions. The largest I use is represented in Fig. 9; it slides

Fig. 10.

Fig. 11.

along the small edges of the rod; the circle do not touch each other, but are yet so close that the exact center of the target can be estimated very exactly; it has no vernier, which, however, could easily be attached.

In order to save a second target, the end of the stadia is shaped as shown in Fig. 12, so making it a stationary target.

Fig. 12.

The colors to be used are best either black and white, or red and white; red has the advantage that the cross wires

can be distinguished on it, which they cannot on a black division. The white ought to have a light yellow shade.

These are the instruments used in the stadia method of topographical surveying. Now, I shall describe the mode and manner of working; I have to make the distinction between work done with the tachometer and work done with the plane-table.

For railroad surveys, with the tachometer, the field party must consist of two engineers, one assistant, two rodmen, who serve at the same time as flagmen and eventually as chainmen, one or two axmen. The engineer in charge of the party, after a general reconnoissance of the country, selects the point upon which the rodmen have to place their stadia; he makes sketches of the general lay of the country in his field book, and numbers the points in his book as taken by the stadia, Goldschmidt's Aneroid will be a good companion for him.

The purpose of the work and the scale of the topographical map—if such is to

be made—determines the number of points to be taken. In railroad work it will generally suffice to take as many points as will enable the engineer to make an intelligent estimate of the amount of earthwork to be done, and to make accordingly changes of the line in his map without going anew into the field. The engineer in charge of the instrument places the same over the initial point, which is chosen so that as large a field as possible can be seen from it, without regarding whether it is in the probable future line. One of the rodmen takes all points to the left, the other all those to the right of the instrument; it is a matter of course that the rodmen must be quite intelligent and well instructed. The assistant has charge of the field book and writes down the readings which the engineer calls out. He also gives the signals to the rodmen as directed by the engineer, and, if time permits, makes the necessary calculations. Some engineers do away with this assistant, but the

employment of one expedites the work to a great extent.

A good and distinct system of signals between engineer and rodmen is very essential. In order to avoid confusion, each tenth point of each rodmen is indicated by them by a signal; also roads, trails, creeks, and similar objects must be marked in a similar manner. Where only one rodman is employed, a whistle or little trumpet will suffice; when two or more rodmen are at work each must have his own style of signal.

Two, or at the most three, rodmen are plenty to keep the engineer and one assistant busy.

In order to avoid mistakes the rodman, who is not sighted at, but has already arrived at his new point, should not put up his staff in correct position before he hears the signal, which allows the other one to move, but must keep it in an inclined position, being ready to place it correctly as soon as required. A well understood code of signals is a very important point.

After a sufficient number of points has been taken, one of the rodmen goes to the engineer in charge, who selects the next point for the instrument, which he must select in reference to a good foresight and in understanding with the engineer on the instrument, as the latter must give the correct grade by setting his telescope at a vertical angle corresponding to the grade the road shall have. Here the rodman uses the target. After such a point has been selected, the instrument is removed to it. Meanwhile, the second rodman has returned to the former instrument point and placed his rod with the target on it; after the engineer has taken his back sight to this point and checked by it his first stadia reading, the rodman comes to him and they proceed as before.

That disturbance in the position of the telescope may be detected and accordingly taken into account, it is advisable to sight at the beginning of the work at a fixed and well marked point, as a house corner or any other well defined

object, and to sight at it again at the end of the work before removing the instrument to the next point. This is a check which ought never to be neglected.

In order to determine the distance between the two instrument points as exactly as possible, and to free the same of all instrumental errors, the readings for those points must be done in both positions of the telescope. The horizontal angles for those points must be read not only with the needle, but also with both verniers; this also ought to be done for the determination of houses, bridges, or other important points. If the instrument has a repeating circle, it is advantageous to place the zero point of the verniers on the zero point of the limb, when the telescope is pointed to the preceding standpoint.

Another precaution, to guard against errors in the distances, is, to determine two or three points in the line about half way between the two stand points, which are sighted at from either one. By this, two measurements of the distance

are obtained, each independent of the other, thus giving a very good check.

The method as described above, of course, allows many variations; each engineer will soon form his own style of working: so, for instance, if good, reliable and intelligent rodmen are to be had, one engineer for the whole party will be sufficient; the progress perhaps will be a little slower, and then besides, the above method has the advantage, that the engineer in charge has an opportunity to make himself thoroughly acquainted with the ground, to make valuable sketches and notes.

The proceeding in a topographical survey for other purposes than railroads, must be a little different, according to the space to be embraced. For railroad survey, only, but a narrow strip of land on both sides of the line is required; but for mining, irrigating and similar purposes the field to be surveyed is of larger extent. Therefore, the following proceeding is advisable. First select a sufficient number of points all over the country to be sur-

veyed, which shall be the future points of the instrument. Select those points so, that at least two other points can be sighted at from each, and that as large an area of ground can be surveyed from it as possible. Secure them with good, solid monuments; then make a triangulation of those points (which operation sometimes may be combined with the actual topographical survey), and determine their heights by a careful leveling. After this proceed with the topographical survey as described before.

This method, of course, is comparatively slow, but gives most satisfactory results, as the work is constantly checked by the triangulation and leveling, which was done independently of the topographical survey; it is the best method for all mining and irrigation enterprises, and, generally speaking, for all undertakings, where permanency of the works is contemplated, and where during the course of the survey some engineering work is in progress. Here, those points, trigonometrically determined and well served

by good monuments, will always serve as reliable points of reference and check; they are of permanent value.

Sometimes it will suffice to determine only a limited number of trigonometrical points, but well selected, so that they can be seen from a great many points in the field to be surveyed; then the instrument points are determined in reference to them. This method will prove most convenient with the plane table, as the points then can be determined by the three point problem.

Although most engineers will make their own blanks of field book, to suit their views and customs, I give here a blank, which has served a good purpose.

No. of instrument, point and height of telescope above datum.	No. of the point sighted at.	Vernier reading on the horizontal limb.		Vertical.	Stadia reading of			Difference.	Reduced distance.	Height of point ± above or below telescope axis.	Height of point above datum.	Remarks.
		Vernier I.	Vernier II.	Angle	Middle wire. (m)	Upper wire. (o)	Lower wire. (u)	$o - u$				
		also Needle reading.										

The stadia method, heretofore described in connection with the tachometer, is still more useful with a planetable.

The alidade is profitably provided with a so-called parallel ruler, which contributes a great deal to the quickness of the work, and—although not correct from a theoretical point of view—is quite exact enough for the work usually required.

The compass box must be a rectangular one to allow a line to be drawn along its edge. By this, the North line is directly drawn upon the paper. After a sheet is filled, or the work finished, it is advisable to draw the scale on the sheet itself, so that the changes of the paper shall not introduce error.

For all further particulars about the plane-table, I refer to the book published by the U. S. Coast Survey on this subject, and, for the adjustments of the instruments, I refer to the catalogues of the different makers.

I will now refer to those topographical maps in which the topography is represented by means of contour lines. To

use the words of Professor L. M. Haupt: "This method of representing topography is vastly superior to any other, as it exhibits exactly the slope of any portion of the ground, gives the elevation of the base of any object within the tract, enables one to make vertical sections in any direction with accuracy from the plot, and so locate roads, paths or other features upon a given grade or at any desired elevation, and furnishes the means of calculating the contents of irregular solids with great precision."

A contour line is a line which connects all points of one and the same height with each other; therefore, their nearness or distance on the maps indicate the steepness or gentleness of the slopes; the nearer together, the steeper, and the more distant from each other, the gentler is the slope of the country represented.

Although not exactly belonging to the subject of this treatise, I will say a few words about railroad locating generally. These are suggested by some remarks of Arthur M. Wellington, C. E., in the in-

troduction to his highly interesting book, "The Economic Theory of Location of Railroads." He says, page 18, and following:

"Another inevitable consequence of such general neglect is that this intricate science of design has been degraded in popular esteem, and even in the minds of engineers, who ought to know better. In former times the ablest engineers gave personal and unremitting attention to the work of location, but we have changed all that at the present day. As soon as a young man has acquired some facility in transit work, and has some glimmering notion that curves and grades are very objectionable evils—or are *not* very objectionable evils, depending on whom he 'ran transit' for—he is forthwith a locating engineer, and he is such in fact in so far as this, that further practice will teach him nothing. For after making one or two surveys he will have mastered the mechanical process of handling a party, and begin to look down on the work of location—because he

knows nothing about it. His work is the dead corpse of location, beginning and ending in the transit. If he is a rising man he will soon find some other young man to take his place in the field, and do the *really important* work, and very probably begin that vicious system of office-location from contour maps which has ruined the alignment of so many railways. Now, all this is especially calamitous, for it is almost a certainty that any one who has not a thorough *theoretical*, as well as practical knowledge of location, will fail entirely to catch the governing features of the region traversed, and find the line which has probably been lying there since time began. The instances are almost innumerable where young men—and old men—of this class have run over and under and across a line of the highest operating value, and turned in a costly and miserable line at last. And the contour-map system does not help this evil even in the hands of a thoroughly capable engineer; for the contour map is simply a

device for doing ill in the office, the simplest part of the work, viz.; the first approximation to the adjustment of the line in detail; and its most effectual office is, to deaden the perceptive faculties of the engineer in charge of the party, and transform him into a mere machine. Of what value is a contour map of an ill-judged line? The truly difficult part of location is the selection of the general route and the final ultimate perfection of its adjustment in detail; and the engineer who can do *this* work well will thank no one for the rude assistance of a contour-map location, made without the detailed familiarity with the ground which is gained by tramping over it. In fact, he will approximate to the detailed alignment quite as well and as rapidly without any such assistance, simply by feeling his way upon the ground, profile in hand, and his party behind him, and guided by a few notes from a rough plot. Nor will such an engineer, if he have a true feeling for the dignity and importance of his work, be content with making

some contour-map guesses to be tested by less skilled subordinates. If he is to interfere in any way and his judgment have any value on paper, it is worth more upon the ground; and there is where he ought to be. He will detect more possibilities while sitting on the fence in apparent idleness than by the longest study of maps, and however long his experience or brilliant his ability, he can at no time in his professional career have more important financial interests depending on a chance inspiration. It should be more generally recognized that the place for the ablest engineers, which money will command, is not in the office or on construction, but in the field at the head of the locating party.

"A large part of this evil is not the fault of engineers, but is due to the fact that the financial loss from bad location is too distant and indirect to excite an amateur's apprehension, and every officer of a railway, from the president down, is an amateur engineer—having all the amateur's fondness for 'meddling and mud

dling' in *unimportant* matters, and all the amateur's reluctance to recognize anything as important which he does not himself understand. The fatuity displayed by the average railway official in this way quite passeth understanding. He will pay lavishly for his attorney's skill in trickery; he will even pay respectably for the manager's skill in dealing with men and with things; but he will neither pay for nor believe in that vastly more needed skill of the engineer, in dealing with abstract physical and mechanical laws, and in determining the financial meaning of their relationship to involved and contradictory facts. For this work he neither seeks for nor will he tolerate anything more than a hand to execute; and the law of supply and demand gives him just what he asks for. Especially is this true in location. On the great majority of railways surveys are entrusted without the slightest uneasiness to the first graduate of the transit who comes to hand: but when he has *completed* his work, and construction is to begin, *then*

we may behold an extraordinary and amusing spectacle. Then we may see half a dozen business men, who probably show some common sense in their own affairs, scouring the country with a lantern to find a constructing engineer of the greatest possible ability at the smallest possible salary—to do what? To pay over the money which is *already spent;* to pare and shave at the cost of work which might have been avoided altogether; to *build* the complicated mechanism for which they have just permitted Thomas, Richard and Henry to make designs and working drawings. This kind of folly has its root in some of the deepest foibles of human nature, and it will probably never be done with altogether; but it is to be hoped that railway companies may more generally appreciate the fact that their road is built and equipped in the brain of their locating engineer—if he has any; and that if his work be ill done, all the engineers in Christendom have done little to remedy his errors, though they execute

his folly for half its proper cost. The truth is, in fact, that ordinary constructive engineering is a much lower branch of professional labor, and makes far less drafts on those qualities of mind which make the engineer. But massive piles of dirt and stone and iron are visible evidences of power which impress the imagination of the wayfaring man as equal evidences of skill, and hence it is not wonderful that the ability of engineers is more generally estimated by the grandeur of the works they have executed than by those which they have avoided."

I quote these words in their entirety, first, because they partly meet with my most hearty consent; and, second, because they are directly contrary to my views and opinions; and, third, because they contain many things which ought to be generally known and considered by everybody interested in this question. I fully agree with Mr. A. Wellington, when he says that the *location* of a railroad is the most important work relating to railroad affairs, which must be con-

stantly and personally attended to by the chief engineer himself, and must not be left to an inferior and inexperienced "transit man." And again, I fully agree with him that the locating engineer and the constructing and building engineer ought to be one and the same person, a person who has experience not only in those two branches of railroad engineering, but also in the operating of a railroad. As Mr. Wellington has so thoroughly and admirably shown in his book, the knowledge of the financial effect of a grade or a curve is the most important in the location of a railroad; and this knowledge can only be derived from actual and personal experience; their effects can be investigated intelligently and successfully only by a man who has a thorough knowledge of the constructing of the road, and why it is so constructed, and not otherwise. The head of operation of a railroad ought to be an engineer, who is not—as he nearly invariably is, I am sorry to say—hampered in his doings by a board of directors

who profess to know everything about managing a railroad, but who, in fact, do not know how to buy and sell sugar and coffee. (That there are some brillant exceptions in respect to these boards of directors ought not to be the cause to make them a rule). This, of course, involves a higher standard of engineers than we usually have; it involves the raising of the engineer profession to the importance it deserves, and finally must and will have. As at present the engineers are situated, it is perfectly shameful; it is inconsistent with the purpose he is here for, and is damaging to the welfare of the enterprise he is engaged for. Here is not the place to treat about this question to any extent, but it is one of vitality to the engineers.

As to Mr. Wellington's views on the contour-plan questions, I have to say the following: If the system of contour maps is carried on and used as it apparently was since Mr. Wellington became acquainted with it, it certainly is a "vicious system." But, if carried out in the right way, it

is certainly the most beneficial system that can be invented. To bring about such an effect, the following condition is essential: the person who makes, or personally and actually superintends, the location of a railroad, must be the same who locates the line in the contour maps; by the survey and the tramping over the ground, he requires a thorough knowledge of it, and has made himself entirely familiar with all its qualities; the contour map, then, is for him a fully intelligible image of the ground, and as it represents a larger field to his eye than he can overlook with one sight in the field, he can judge more intelligently about the relations of distant parts to each other; he can at once decide the effect a change at any point will have on any other point. With what right Mr. Wellington says, "for the contour map is simply a device for doing ill in the office, the simplest part of the work, viz., the first approximation to the adjustment of the line in detail, and its most effectual office is to deaden the

perceptive faculties of the engineer in charge of the party, and transform him into a mere machine," I cannot explain otherwise than that he has not had much experience with the system, and that he did not get on the right side of it. The contour map is just like a relievo of the ground, and enables the engineer to work in it as the sculptor works in his clay; he can mould in it as the circumstances require it. The engineer, who has a thorough knowledge of the ground, and locates a railroad on a contour map, in comparison to the engineer who locates the railroad but in the field, where his field of view is but limited, is like the general who leads a battle from an elevated standpoint, to the officer who has charge of only one wing of the army, being situated so that his eye can embrace but the small space occupied by his own regiment or battalion. Now, as a change of any part of a railroad line—which is a continuous, uninterrupted line—affects always some other part of it, it is necessary to investigate at once the effects the

change will have on the whole line. If there is no contour map it is necessary to locate a shorter or longer part of the line anew, which again may prove not advantageous, so necessitating a third location of this part of the road. This is the cause of great delay and expense. But when there is a contour map, such expenses can be avoided. The engineer, who is familiar with the ground, locates in his contour map the new line, calculates by means of the same map the amount of earthwork to be done, finds, perhaps, that this new line is not an improvement, tries another one, calculates again its cost, and so on until he finally finds the best line. And this is all done with but a slight expense. This, of course, always supposes that the contour map is a correct one, and not on too small a scale. (1.1000 or 1.500 are the most practical scales.) I will give shortly the account of a location with this system, as actually carried out by myself. I shall omit the account of the survey for the contour map, and shall suppose the latter to have

been made. It was constructed in a scale of 1.500, a scale which allows the smallest unevenness which would influence the location of railroad to be expressed. The base line, on which the survey was founded, was approximately the future railroad line, but, of course, without curves. The first thing was to lay down the curves in the map, which were not staked out in the field, and to calculate the grade for about every 100 feet, then the so-called "intersecting curve" was constructed in the plan. This is the line, where a plane laid through the imaginary height of railroad intersects the ground; it represents to the eye at one glance approximately the point where too much cutting or too much filling would be necessitated, hence where the line should be changed. Where this was the case, the line was moved until it laid in about the center of the "intersecting curve," *i. e.*, so that on each side of the line about an equal part of the curve was lying. This could be found without much calculation of cross

sections. When the probable best line was found the sections were constructed and calculated, which were easily and quickly constructed from the map by assistants, the one reading the distances and heights from the contour map, and the other drawing the cross section on profile paper equally divided each way. The areas of the sections and also the cubic contents were found by means of the planimeter, the latter in this way: Draw in the center of the paper a horizontal line which shall be the axis of the ordinates, set off on it all distances of the cross sections, and erect in these points verticals; where cutting is draw verticals above the line, and where filling, below; then set off on each of those verticals the respective area of the cross section (the areas represented by length) and connect the end points of these verticals with each other, by a continual and smooth curve; the scale for the areas, of course, may be another one than that of the distances. Then find the areas of the

figures enclosed by those curves and the horizontal center line with the planimeter. These areas will be the cubic contents.

These few remarks will suffice to show how useful the contour maps may be when rightly used. I shall now describe how the topographical map is made from the data derived from the tachometer. The first thing to be done is to lay down the base line, or the line which connects the different instrument points with each other, which is done by the common method of latitude and departure, or sines and cosines. The intermediate points are laid down from each point by means of a protractor, which is divided into half degrees, and has on its straight edge two scales with a common zero point, which lies in the vertical drawn through the line of 90°. The graduation of the protractor is numbered twice, once from 0° to 180°, and then from 180° to 360°. The numbers run in the direction opposite to that of the instrument. The center point of the protractor

is secured by a little horn plate with a hole in its center; this is brought over the station point and a needle put into it, so that the protractor can be turned around it as a center point. One person reads the angles and distances from the field notes (which have been completed first in regard to reduce distance in height), the other person first places the protractor so that the zero line coincides with the north line, then turns the same as much as the angle requires, and marks the distances by means of the scale and fine needle on the map. The scale of the protractor, of course, must be the scale of the map. After the point is marked down, the height as given from the field notes is written near it. After all points are laid down in this manner, the contour lines must be drawn, which can be done in many different ways; it should be done by the engineer who has charge of the field party, because he is the most familiar with the ground.

According to my experience, the best and quickest way is the following: Use paper which is divided into squares, with sides of one-tenth of an inch length; then draw a profile through the two points between which the contour lines shall be constructed. The intersections of the horizontal lines with this line will be the points of the contours, and their distance from the center vertical line will be their horizontal distance.

The curves must be drawn with great care, and full understanding of the ground: the construction is a problem of descriptive geometry, and requires great attention.

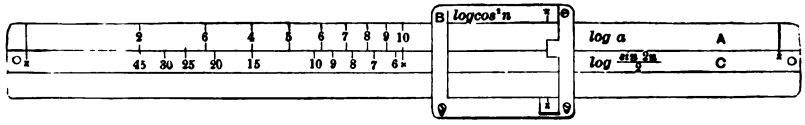
The points actually obtained should not be rubbed out after the contour lines have been constructed, but they must be preserved by a little black point, and the number indicating the height also in black. The contour lines should be drawn either with burnt sienna or with green; their numbers must be written on them at many points with the same color. Each fifth or tenth curve should be drawn

in a little different manner from the other —for instance, dotted or stronger; this contributes a great deal to the distinctness of the plan. All other details of the map should be marked black with the conventional topographical signs. The steepness of the ground, the scale of the map and the purpose of the work, determine in which heights the contour lines shall be drawn, whether for each foot or for each 3, 5, 10, 20, or 100 feet.

THE SLIDE RULE.

It would be very tedious and slow to calculate for each point the respective values according to the formula, as above given for the distances and heights. There are several tables published which, with two arguments, give the respective values (one is calculated by Alfred Noble and W. T. Casgrain, assistants U. S. Engineer office at Milwaulkee), but the best device is a slide rule, which was first constructed by the Swiss Engineer Eschman, and afterwards improved by Professor Wild in Zurich.

Fig.13.



I suppose the theory and use of the common slide rule is known to the reader (if not I refer him to my pamphlet on this subject).

The slide rule as used in topographical surveys consists of a ruler A, a slide C, and a coulisse B.

The ruler has on its upper part four equal scales, each of which is a logarithmic scale of the common numbers. The scales commence with the number one, as the logarithm of $1 = 0$; the space between the numbers 1 and 2 is divided into 50 parts; that between 2 and 3, and 3 and 4 and 5 into 20 parts each, and that between 5 and 6, 6 and 7, 7 and 8, 8 and 9, 9 and 10 (or 1 of the following scale) into 10 parts each; hence the scales read as follows, commencing on the left 1, 1.02, 1.04, 1.06, ... 1.98; 2. 2.05, 2.10. .....4.95; 5.00, 5.10, 5.20, 5.30, ... 9.90, 10.00. With increasing numbers the divisions become smaller, as differences between their logarithms become smaller. The values between the divisions must be estimated. The numbers

indicated on the scales can stand either for the numbers themselves, or they may stand for any decimal value of them: thus, 1 stands for 1, 10, 100, 1000, etc., or 0. 1, 0.01, 0.001, 0.0001, etc.; 2 stands for 2,20, 2.200, 2.000, etc., or 0.2, 0.02, 0.002, 0.0002, etc. It is a matter of course that the value given to one number of the scale influences the whole. It is practicable to give the first scales to the left, the value of from 10 to 100, and the second of from 100 to 1000.

On the coulisse B. there is the scale of log. cos. n^2 (see formula 3); this scale counts from the right to the left, as $\cos.^2 n$ is always smaller than one, their logarithms are therefore negative. The space from

0 to 10 is equal to the log. $\cos.^2 10°$,

that from

0 to 20 is equal to the log. $\cos.^2 20°$,

0 to 40 is equal to the log. $\cos.^2 40°$.

The first part of the space 0 to 10 stands for log. $\cos.^2 4°$, the second for log. $\cos.^2 6°$, and the third for log. $\cos.^2 8°$. The

part between 10 and 20 stands for each two degrees, and those between 20 and 40 for each degree.

The scale on coulisse B in connection with the scale on A, are used for calculating the distance; it is:

Log. $d = \log. \quad (a \cos.^2 n) = \log. a + \log. \cos^2 n$ log. a (the logarithm of the stadia reading) is given on scale A, log. $\cos._2 n$ on scale B.

Place the point 0 of the coulisse B above the stadia reading on the scale A (on above the stadia reading plus $1.5p$), and look which number in the latter scale stands below the vertical angle of scale B; this will be the horizontal distance.

Examples: Stadia reading $a = 2.48'$, $p = 12''$, $n = 5°20'$; place 0 of scale B above 249.6 $(ak + c)$ of scale A, and read under $5°20'$ the reduced distance estimated to $248'$; if the angle were

10°, D would be = 242′,
20°, D " = 221′,
30°, D " = 187.7′,
40°, D " = 146.4′, etc.

From this instance it can be seen that for smaller angles the result, as given by the slide rule, is not as exact as for greater angle, but still exact enough for practical purposes.

On the slide C there is the scale of $\log. \frac{\sin^2 n}{2}$ [see formula (5)]. It commences with the value for 35 minutes at the right hand end, and the graduations from 1 to 3 stand for each two minutes,

3— 5	"	"	5	"
5—10	"	"	10	"
10—20	"	"	20	"
20—30	"	"	30	"
30—40	"	"	1°	"

from 40°—50° there are no small subdivisions.

Formula (5) is

$$\log. Q = \log. ak + \log. \frac{\sin 2n}{2};$$

therefore, place the line for the vertical angle on the scale C under the stadia reading of scale A_3 or A_4 and find above the left index (the line with the star) the height. In case the left index falls be-

yond the scale, the center index or the right one can be used, but it must be considered that the center one gives ten times, and the right one hundred times the reading of the left index.

For the number of places of the height we have the following rule: If the height be found in the same scale as the distance (or the value ak) is taken and the left index be used, the height has as many places as the distance; but if, in the same case, the right index be used, it has two places less than the distances, and if the center index is used it has one place less than the distance; if the height be found in the preceding scale and with the left index, it has one place less, and if, in the same case, the center index be used it has two places less, and if the height be found in the following scale with the right index, it has one place less, and if with the center index it has just as much as the distance. In the following table z stands for the number of places of (ak).

Scale in which height is found.	Which index used.	Number of places of height.	ak	n	Q
Same as distance.....	left	z	400	18°	118. 8′
" "	center	z — 1	2400	5°	208. 0′
" "	right	z — 2	2400	37′ 48″	26.16′
Preceding.	left	z — 1	900	54′	14. 6′
"	center	z — 2	1750	2° 42′	82. 5′
Following.....	center	z	34.7	27°	14.05′
"	right	z — 1	9	54′	0.141′

For smaller angles than 35 minutes the angle must be multiplied (perhaps by 10), and the result divided by the same number, which safely can be done, as for small angles the sine is nearly equal to the arc. If we had, for instance, $n = 0°6'$, we would take $6 \times 10 = 60'$ or $1°$, and place this angle below the stadia reading and divide then the result by 10. Example: $a = 3.45$, $n = 20'$; place the angle $10 + 20' = 3°20'$ below 345, find

$$Q = \frac{20.00}{10} = 2.00$$

(the exact result is 2.05.)

On the lower edge of the slide rule, there is yet another scale, which is used for the reduction on account of refraction and curvature of the earth in greatly extended topographical surveys. For this scale, the lowest index, which corresponds with the others, is to be used in this way: place the index of the coulisse B over the distance in scale A, and find the correction under the lowest index on the scale of the lower index. These

corrections are in the metric system. For instance:

$D = 500^m$, Correction $= 0.017^m$,
$D = 1000^m$, " $= 0.068^m$,
$D = 1500^m$, " $= 0.16^m$,
$D = 2000^m$, " $= 0.26^m$,

etc., which correction is to be subtracted from heights.

NEW METHODS

IN

TOPOGRAPHICAL SURVEYING.

BY

PROF. A. S. HARDY.

NEW METHODS

IN TOPOGRAPHICAL SURVEYING.

WHILE in Paris, during the winter of 1874, the attention of the writer was called to the extensive application of photography to topographical engineering, as practised by the French engineers. This fact was pointed out in the Reports of the U. S. Commissioners to the Paris Exposition of 1867,* with a brief description of the principle under which the process was conducted; but so far as the writer is aware, the method has received no practical application in this country, nor the attention which is its due. The value of topographical maps, especially in railroad surveys, is too well known to be insisted upon, and the aid they have rendered in France, especially, is very notable. Any process by which

* Vol. V. Report on Photographic Apparatus, p. 11.

the time and thus expense, of a topographical survey may be reduced, has therefore a peculiar value in this country where the magnitude of the work has proved, as it were, a dead weight to any extended project of this nature. The photographic process, for example, would be invaluable in the projected survey of the State of Massachusetts, and will amply repay the brief study which its novelty demands. It has long been the practice in hydrographic and topographical surveys to make sketches of the shores or landscapes, on which are written, near prominent points, their angular distances measured by an instrument. This was extensively practised in the hydrographic surveys of the West coast of France, and notably also in the surveys made during the voyage of the "Bonite." In the report of the Abyssinian Commission made to the *Académie des Sciences* in 1846, M. Arago urged the adoption of panoramic views, with the angular distances between prominent points (one of which should be exactly

located) inscribed thereon, as a prevention against errors and a precious source of reference for all time. Even as far back as 1802, a Commission had been appointed by the French War Department to study this subject, and its report* contains the following remark: "The Commission believes it is always useful and often necessary, in topography, as in the other arts, to add to the horizontal projection, or plan which constitutes the map, a vertical projection or perspective, and desires that when possible this may never be neglected, even when at the time its utility is not apparent."

The first systematic study of this subject was made by M. Laussedat of the Engineers. For this purpose, he employed, in 1854, Wollaston's Camera Lucida, under a slightly modified form to avoid parallax. Subsequently, in 1861-2, this study was extended to the Camera Obscura. With the assistance of Capt. Ducrot and others, M. Laussedat used the process to be described in numerous

* Memorial du Depot de la Guerre. Vol. II., p. 11.

extended surveys, and it is to the courtesy of the latter, now Colonel of Engineers, that the writer is indebted for many details which are the fruit of experience alone.

As evidencing the economy of this method in time, reference may be made to the work done in Savoy and the Vosges. In the former department, one survey of 18 days' field work sufficed for 30,000 acres, contour lines being mapped 5 meters apart, giving 5 months' office work. In another case, 110 proofs were taken for 20,000 acres, the field work consuming but 15 days. These examples are taken at random from among many instances to show the relative time required by this and the usual method. This will depend somewhat, of course, upon the character of the country, but M. Laussedat has not found that it requires more than one-third that by the ordinary triangulative and often less. In the field work alone the economy is very apparent. The instrument employed is a combination of the camera and

theodolite. The camera proper carries on its front face the usual objective, mounted in a sliding tube, so that the focal plane may be made coincidental with the sensitive plate at the rear of the chamber, and this tube is provided with the usual diaphragm to insure the distinctness of the images. A cover similar to that of the telescope excludes the light, but should slide easily on the objective without disturbing the instrument when leveled. Once focussed, the position of the tube may be marked, as it will not be necessary in landscape views to readjust it, as for near and distant objects. The grooves in which the slide containing the sensitive plate moves should be constructed with care, so that the latter may exactly occupy the focal plane. Within the chamber are placed four fine needles, one in the middle of each side near the slide, destined to intercept the light, thus marking on the proofs four points, which joined, give a horizontal and vertical line through the center of the

field of view, whose use will be shortly noticed.

The chamber is supported in the usual way by two cylinders, one solid and fixed to a tripod with leveling screws, the other, hollow and enclosing it, is fixed to the chamber. The chamber may thus be revolved about the vertical axis with the hollow cylinder which carries a vernier reaching a graduated limb fixed to the inner axis. An 8-inch limb with a minute reaching vernier is sufficiently accurate for all operations which are to be graphically reprinted.

On one side of the chamber is a telescope and level. This telescope has a motion about a horizontal axis, and carries in its revolution a vernier reaching a vertical limb fixed to the side of the chamber. The plate on which this arc is engraved is one piece with the axis, which projects simply far enough to permit the vertical motion of the telescope. This apparatus, as well as the objective, may be dismounted for packing in a separate box as usual, and a counterpoise

on the opposite side of the chamber insures stability when mounted. The adjustments of this instrument are obvious:

1st. The axis of rotation of the telescope must be vertical. In the instrument seen by the writer there were but three leveling screws, and the horizontal limb was so constructed that when the zeros of both verniers were at the zeros of their respective limbs, the level was parallel to a line, joining two of the screws. The adjustment was then readily made with the leveling screws and tangent screw to the telescope. This construction, common to French instruments, is of course unessential.

2d. The line of collimation of the telescope, which is provided with both cross and stadia hairs, is effected as usual.

3d. The optic axis is made horizontal as in the ordinary geodesic instruments, and the reading of the vernier after adjustment is the error of collimation, to be added or subtracted, according to its

sign, to the vertical angles subsequently taken. Two important conditions must be fulfilled by the maker: (*a*) When leveled, the axis of the telescope and the optic axis must be at the same height and thus describe one and the same horizontal plane during the chamber's revolutions; (*b*) The slide at the rear of the chamber, when in position, should be vertical and perpendicular to those axes.

It is thus seen that the instrument differs from those ordinarily in use only in a few details dependent upon their combination, and its use requires a knowledge of only the simplest principles of scenographic projection. Indeed, the proofs are themselves conical projections, the optic center of the line, which is the vertex of the cone, being the point of sight, and, as in landscape views, the objects represented are so far distant as to have their images formed on the same focal plane, the distance of the point of sight remains constant.

Thus let O be the optic center of the objective, its axis O P being horizontal,

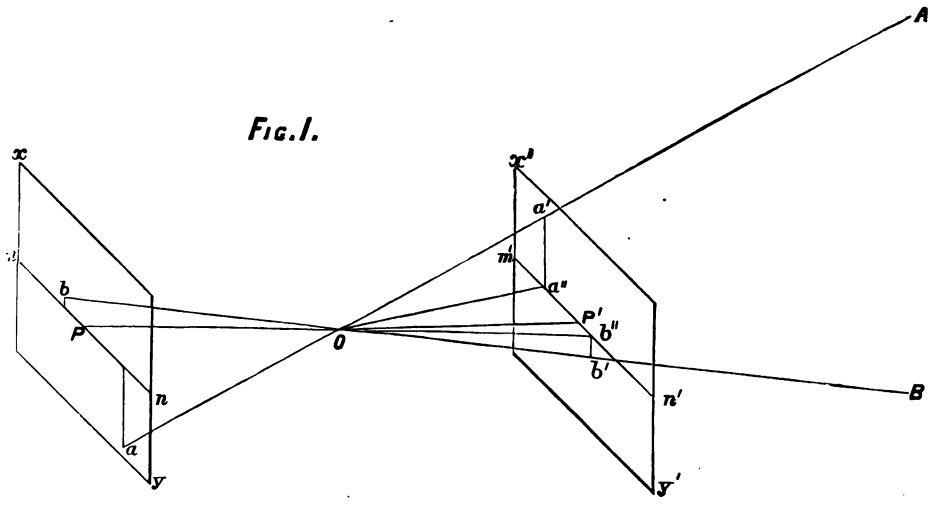


FIG. I.
x
x'
A
a'
m'
a''
b
P
P'
b''
0
b'
B
n
n'
a
y
y'

and $x\ y$ the glass slide at the rear of the camera, occupying the focal plane. Then O is the point of sight, P the principal point, $m\ n$ the horizon, and any two objects as A and B will appear at a and b. If a glass $x'\ y'$ were placed between the objective and the landscape, and at a distance from O equal to O P, the representation would be similar to that on $x\ y$, and will in part correspond to a *positive* proof.

If the perpendiculars be let fall from a' and b' upon the horizon $m'\ n'$, and their feet joined with O, then will oa'' and ob'' be the projections on the plane of the horizon of the visual rays O A and O B, the angle $a''\ ob''$ will be the angular distance between A and B reduced to the horizon, while the angles $a'\ oa''$ and $b'\ ob''$ are the angles of elevation or depression of objects above and below the horizon. All points of the landscape at the same level as O will appear on the horizon, the curvature of the earth, unimportant in such operations, being neglected. The proof is thus a conical projection whose

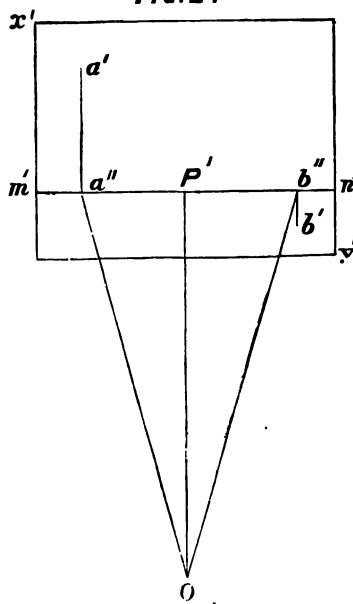
point of sight is the center of admission of the lens, and the distance of the point of sight from the plane of the picture is the principal focal distance.

If then $x'\ y'$ be a photographic view on which the position of the principal point P′ and the horizon $m'\ n'$ is known, as well as the principal focal distance of the lens, let the plane of the horizon be revolved about $m'\ n'$ until coincident with the plane of the picture. O will be found at a distance from $m'\ n'$ = principal focal, and is the revolved position of the point of sight. Join the foot of the perpendiculars $a'\ a''$ and $b'\ b''$ with O, then $a'\ ob''$ will be the horizontal angle between the objects having a' and b' for their images, *i. e.*, the angle usually measured in the field. Finally, the vertical angle of any object as that whose image is a', is obtained by the ratio $\frac{a'\ o''}{oa''}$, the trigonometrical tangent.

Both the vertical and horizontal angles are thus determined from the vicus.

Suppose, now, a base line ab measured

FIG. 2.



in the field, as also two angles *c b a* and *c a b* on any prominent object as *c*, and that *a b c* be the plot to any convenient scale. Having two angles and a side, the distances *a c* and *b c* may be computed.

Suppose also two views $x'\ y' + x\ y$, taking one from a and one from b, and both containing the object c at c' and c, respectively. Having let fall the perpendiculars $c'\ c'$ and $c_1\ c_2$, let the proofs be revolved about their horizons into the plane of the picture, and the points of sight placed at a and b respectively. Join c'' with a and c_2 with b, and revolve each wire about a and b respectively, till $c''\ a$ and $c_2\ b$ pass through the point c previously determined. In this position any object on both views may be located on the plan. Thus, one whose images are f' and f_1 will be found at f, the intersection of two lines af'' and bf_2 drawn from the points of sight to the foot of the perpendiculars $f'\ f''$ and $f_1\ f_2$. It is thus evident that a great number of points may be determined without further direct measurement. The plot is verified as usual in the methods of intersections. A third view, $x''\ y''$, taken at any point as d, and containing the object c, is placed in a position as before, so that the line dc_4 passes through c. A line joining

d and f_4 should pass through f. Once verified in this manner, all views containing objects thus fixed can be placed in position. M. Laussedat, however, finds it preferable to measure each base, and to take either one or two angles to fix the position of each proof. Usually the angles between the basis and the angle between base and principal point are measured. All other objects which are to be represented on the plot are located from the proofs.

To determine the height of f'', for example, suppose the focal distance $1^{ft}.5$, and let $af'' = 1^{ft}.55$; $f''f = 0^{ft}.05$, and $af = 0^{ft}.55$, and the scale be $\frac{1}{8000}$. Since $f'f''$ is the apparent height of f' at the distance af'', and $af \times 8000$ its true distance from the proportion

$$af'' : af \times 8007 :: f'f'' : x,$$

we have

$$x = af \times 8000 \frac{f'f''}{af''} = .55 \times 8000\,1.55 = 141^{ft}.9.$$

Hence the height of any object above the

extremity of a base is found by multiplying the tang. of the angle of elevation (to a radious equal to the distance of the station from the foot of the perpendicular through the object) by the true distance as found on the plot. To this product the height of the instrument must, of course, be added.

The heights are also verified by performing this operation with reference to two stations *a* and *b*, whose difference of line is known. It is not necessary that this difference be measured in the field, since any difference in line between two sections will be indicated on the proofs by a change in the position of the horizon, and may be therefrom determined.

This horizon is indicated on the picture by the shadows of the needles already mentioned as placed within the camera. These needles are adjusted by the maker, but are held in small pieces moving in grooves, so that their readjustment is always possible. To effect this it is only to be remembered that when the instrument is in adjustment and re-

volved, the axis of the camera and telescope describe the horizon, so that if, during this motion, the intersection of the cross-hairs be fixed on any object its image on the glass slide will fix one point of the horizon, and by turning the instrument to the right and left till the object is brought to the edges of the slide, two points may be there marked, and the shadows of the needles should fall on the line which points them. The position of the needles giving a vertical through the center may be verified on the positive proof. Except in case of accident, this adjustment, if made by the maker, need not be repeated.

If, as is usual, the objective is fixed in the middle of the front of the camera, the horizon will divide the proof into two equal portions. M. Savary has modified this arrangement by making it movable in two vertical grooves, one of which is graduated to permit the measurement of the displacement. In very mountainous districts the position of the horizon on the proofs may thus be changed. When

the point of sight is above objects whose images do not fall within the limits of the proof, the horizon should be lowered, and *vice versa*. The line marked by the needles will then indicate a parallel to the horizon, which may be drawn parallel to this line and at a distance from it given by the graduation near the objective, whose zero, of course, corresponds to that position in which the horizon is given by the needles. As the image is reversed, when the objective is raised the horizon must be drawn below, and *vice versa*. Ordinarily the views do not cover all the field, and this simple expedient permits the increase of the field of view in certain cases, without increasing inconveniently the dimensions of the apparatus.

The distance of the point of sight from the picture, *i. e.*, the focal distance of the objective may be found from the triangle $a''\ P'\ O$ (a'' being any object on the horizon), by the formula

$$OP = a''\ P'\ \cot a''\ OP',$$

a'' P'' being measured on the proof, and the angle a'' OP' by the horizontal motion.

The question has probably already occurred to the reader, to what extent does spherical aberration prove a source of error in the use of the camera obscura? For objects distant from the center of the field of view will not have their images formed *exactly* in the principal focal plane. The very able researches* of Col. Laussedat on this subject, published in 1864, show that in the clearest manner for all ordinary cases where the apparent heights of objects above the horizon on the view are small, the vertical component of the angular deviation (which is the same in every direction, everything being symmetrical about the optic axis) may be neglected. So that, except in rare cases, the trigonometrical tangent, already given, is taken as the measure of vertical angles. The horizontal component, however, can not be neglected. with a simple achromatic objective of $0^{m}.081$ diameter,

* Memorial de l'Officer du Genie, No. 17, p. 273.

and a focal distance of about 0 .5, and a diaphragm of $0^m.015$ opening, $C^m.077$ in front of the nearest lens surface, the focal distance was found to differ slightly with the position of the point a''. Thus, for a point $0^m.005$ from the principal point, the focal distance was $0^m.505$, but for a point near the border of the field of view, $0^m.1476$ from the principal point it was $0^m.500$. That is, the focal distance diminished as the point from which it was calculated receded from the principal point. Evidently, then, if in the construction of the plan the focal distance was used, as found by a point near the center, the error would increase as the instrument was turned, and in 360° would reach, in the above case 4°. If, however, we use for focal distance that calculated from a point near the border of the field, the error does not multiply; near the principal point and borders it is altogether insignificant, and midway between is a maximum, where in the above case it would not depress 5 minutes. Without corrections, therefore, an exactness is

obtained by this precaution more than sufficient for graphic constructions.

After what has preceded, the following *résumé* will be clearly understood:

FIELD WORK.

This includes first the measurement of the bases and angles. The notes are kept in five columns, in which are recorded: 1° the positions of their stations; 2° the lengths of the bases; 3° their included angles; 4° an angle measured between the base and any point in the field of view, serving to fix the position of the views on the plan. (These may be the principal points). 5° remarks.

The base may be measured by the chain. With the 12½-inch telescope seen by the writer, and adopted as a good size for a camera whose horizontal dimensions are $16'' \times 18''$, bases not excluding 1,000 feet, may be measured to within less than $\frac{1}{200}$th by the stadia. The stations are best chosen on the borders of the survey on dominant points,

but central ones may be necessary, and in an extensive survey are selected very much as in ordinary triangulation, those being the most advantageous which are sufficiently elevated to unmask more distant objects. Stations near together are to be avoided, as the bases are thus short as compared with the lines of intersection which fix the objects on the plan, and therefore intersect under an acute angle. Should this be unavoidable, a simple method of avoiding inaccuracy will be indicated in the description of the office work. The photographic operations do not need description here. It may, however, be said that the French engineers prefer the use of paper to glass, which is fragile and heavy in extended surveys. The positions thus obtained, though less distant usually than those obtained with glass, are sufficiently so.

Finally, in certain localities, as among buildings or in depressions, slight sketches will complete the details and obviate a multiplication of views.

Inasmuch as the focal distance is the scale of the views, it bears a relation to the scale of the plan. M. Laussedat states that experience has shown a focal distance of $0^{m}.5$ best adapted for scales between $\frac{1}{2000}$ and $\frac{1}{10000}$. The distance at which one may operate also depends upon the scale. Suppose the scale chosen is $\frac{1}{3000}$, then a focal distance of $0^{m}.5$ will represent on the plan 1,500 meters, and points at a much greater distance from the station will not be obtained with the desired precision. Were the scale $\frac{1}{8000}$, however, the operations could be conducted at 4,000 meters. The bases are just plotted with the protractor, and at each station is laid off the angles, taken from the notes, between the base and principal points of the several views. On each of these lines is laid off the focal distance, and at their extremities a perpendicular drawn, which is the trace of the plan of the picture.

A distant position serves for the deter-

mination of the focal distance. With this distance for a radius, the trigonometrical tangents of 1°, 2°, 3°.....15° are calculated (a field of 30° giving the best results) and these distances are laid off each way from the center of a horizontal line on a separate piece of paper. Through the points of division perpendiculars are then erected, and the descents of the same angles to the same radius computed, they being the length of the visual rays between any object and the point of sight, reduced to the horizon. Finally parallels are drawn to the horizon at equal distances apart. This diagram is then transferred to all the proofs taken with the same objective, taking care to make the coincidence between the horizontal line and the vertical through the center, and the horizontal and vertical line through the principal point on the proof exact. Every point is thus referred to the horizon and vertical line, and the length of its visual ray reduced to the horizon is known.

To fix any point on the plan, its hori-

zontal co-ordinates on two proofs are transferred by the dividers to the horizons on the plan, and their extremities joined with the proper points of sight. To determine the heights we have already deduced the formula

$$z = sd\frac{h}{d'} \text{ in which}$$

x = true height sought, h = apparent height $f'\ f''$ (Fig. III.), sd = true distance af, s being the denomination of the scale, and d' = the apparent distance af'', or the visual ray reduced to the horizon.

As in this formula d' is the secant exactly calculated and s is given, any error that may arise will be due to d and h, the former being measured on the plan, and the latter on the proof. Both of the errors due to d and h will then be multiplied by the fraction $\frac{s}{d'}$ and therefore are proportional to the scale and focal distance. Furthermore, any error in h, being multiplied by d, will be greater as the object is further off. Views should then be taken as near as possible to

objects where heights are desirable. For distances less than 500 yards the error will not exceed one foot with an objective of 1.^{t} 64 focal distance. For extended surveys in which the contour lines are 10′, 15′, or more feet apart, all desirable accuracy is obtained with the above precaution, while, as already shown, an accuracy more than sufficient for graphic construction is obtained for the plan.

The office leveling notes are kept in eight columns as follows: In the

1°. Designation of the points whose heights are sought.

2°. d'—the calculated descent or visual ray.

3°. sd—the true distance taken from the plan.

4°. $\pm h$—the apparent height, taken from the proof.

5°. $\pm x$—the real height calculated from the formula.

6°. The height above the plane of reference of the stations to which objects are referred.

7°. Absolute height above plane of reference.

8°. Remarks.

In operating upon bases small as compared with the visual rays, construct, on tracing paper, for each station a few horizontal angles for objects as distinct and far apart as possible, and fix these constructions on the plan as usual. Should the intersections not prove perfect, and if the angles have been carefully constructed, by very slightly moving the paper in succession, their position after a few trials may be completely rectified. The principal lines may then be marked lightly on the plot in pencil.

The leading operations and principles of the method have now been described. Both M. M. Ducrot and Laussedat have compared at different times the results of this method with those obtained by an ordinary survey on the same ground, and found them remarkably exact, even on very difficult ground. The advantages of this application of photography are evident. No sketch can compare in com-

pleteness or exactness with photographic views, and by no other means yet known can the time and labor of a topographical survey be thus abridged. In proportion as the survey is small and the greatest possible accuracy requisite, this method loses its superiority. But for larger surveys, its advantages are unquestionable, and in all cases may be made a valuable source of contribution to those details which would otherwise demand a long and tedious direct observation, and the photographs constitute a series of notes good for all future reference. As briefly exposed in this paper, it is regarded as the last used on this subject by French engineers, and in view of the probable increase in topographical surveying in this country deserves the attention of our own.

THE "GEOMETRY OF POSITION"

APPLIED TO

SURVEYING.

BY

JOHN B. McMASTER, C. E.

THE

Geometry of Position Applied to Surveying.

INTRODUCTORY.

I. I PURPOSE to set forth, as briefly as possible, a few results obtained from the application of the "*Geometry of Position*" to the solution of such problems in surveying as are of every-day occurrence, believing that the results thus obtained will not prove altogether uninteresting or unprofitable. As the *Geometry of Position*, however, is a branch of mathematics, scarcely known even by name in this country, some statement of its peculiar character and chief merit seems quite in place by way of preface.

That such a statement can be justified on such grounds, is, to say the least, a matter both of regret and surprise. Of regret, that so simple, so beautiful, so eminently useful a branch of mathematics

has been suffered to remain so long unheeded. Of surprise, that in spite of the relation in which Geometry stands to all other branches of mathematics, a most important advance in Geometry is quite unknown; that in spite of the efforts so persistently made to simplify and reduce all mathematical processes, a most effective agent for this purpose is yet unused. Nothing perhaps is more characteristic of the present state of the mathematical sciences than the simplicity of their solutions. It seems, indeed, as if that inventive spirit which in the industrial arts has led to the production of numberless labor-saving machines, has invaded even mathematics and led to the production of all manner of labor-saving processes of solving long and difficult problems, till now it is possible to solve graphically, with the ruler and pencil, any problem from a proposition in the rule of three, to the determination of strains in the parts of the most complicated engineering structure. In the industrial arts such labor-saving machines are said to be the

direct result of a lack of skilled manual laborers: perhaps in mathematics such labor-saving processes may, to some extent, be the direct result of a lack of skilled intellectual labors. However this may be, the Geometry of Position, though capable of accomplishing much in the above mentioned respect, has as yet been turned to no use.

As a branch of Geometry it is peculiar in all respects. If the story that has come come down to us from the time of Thales is deserving of any credit, the science of Geometry arose from the efforts of the Egyptian priests to restore the land-marks and boundaries washed away by the yearly inundations of the Nile, and to determine the areas covered by the fertilizing waters; a statement which the Greek name Geometry—"*Land-Measuring*" tends not a little to confirm. But whatever may have been its origin this much is certain, that Geometry as a science first appeared in the valley of the Nile, and that among the Egyptians it held very much the same place surveying holds with the nations of

to-day. At the very beginning, therefore, Geometry became associated with the idea of measure, and so firmly has this been clung to, that ever since, in whatever form the science has appeared, whether as Trigonometry or Analytical Geometry, magnitude or quantity has formed the basis. It is now universally conceded that measure is not an essential element of Geometry; and that while there is a *Geometry of Magnitude* or *Quantity* there is also a *Geometry of Figure*. Almost within our own day the science has been divided into the "*Old Geometry*" or "*Geometry of Measure*" and "*Modern Geometry*" or "*Geometry of Figure*" under which is to be placed the "*Geometry of Position.*"

II. THE GEOMETRY OF POSITION differs essentially from the Old Geometry or Geometry of Measure in three particulars; in the simplicity and paucity of its elements; in the total absence of the idea of measure and all metrical relations; and in the great generality and comprehensiveness of its principles and problems.

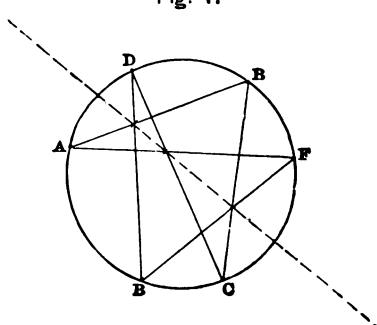
The fundamental elements of the ancient geometry are, the point, the line, the plane, the angle, the circle, solid bodies, surfaces of revolution and the long array of triangles, rectangles and polygons. The fundamental elements of the *Geometry of Position* are the point and line. The most marked peculiarity, however, is the total absence of all metrical relations. In the old geometry the metrical relations of the parts of the figure are never for a moment lost sight of. It is the length of some line or the bisection of some angles, the equality or similarity of some figures, the value of the square of some side that is to be demonstrated. It is a proposition on the intersection of medial lines, on the measure of inscribed angles; on the ratio of homologous sides; on areas, on volumes, on circumferences, on perimeters. The Geometry of Position, on the other hand, takes no account of measure, either angular or lineal. In none of its problems is there to be found any mention of perpendicular or oblique lines, of angles,

of areas, or of volumes. It has nothing to do with triangles, whether right or oblique, isosceles or scalene; nothing to do with rectangles or parallelograms, or with regular or irregular polygons. No line is ever bisected: no angle is ever read. The student therefore, who, familiar with the principles and problems of the Old Geometry, enters on the study of the Geometry of Position, finds himself, so to say, at the beginning of a new science. Theorems and axioms, corollaries and scholiums, which he has long looked on as the very frame work of geometry, are utterly abandoned, and, without considering the length of a single line or the measure of a single angle, he enters on the solution of problems of the utmost generality and comprehensiveness. In the Old Geometry of measure the majority of propositions are necessarily limited in the scope of their application; the conditions on which their demonstration depends are particular, rather than general. Change the length or inclination of a line, alter the measure of a

single angle, and the proposition falls to the ground. Propositions that are true of right-angle triangles are not true of oblique-angled triangles. Propositions true of equilateral are not true of scalene triangles. In the *Geometry of Position* whatever is true of *one* figure of three sides is equally true of *every* figure of three sides, no matter how long or short the sides may be, or how various their inclination to each other. In the Geometry of Measure, again, what is true of the circle is not necessarily true of the ellipse, the hyperbola, and the parabola, but such is the comprehensiveness of the Geometry of Position that *every* proposition true for the circle is true for the ellipse, the parabola, the hyperbola, in short, for every curve of the second order. The length of the radius, the position of the foci, the length of axis is never for a moment considered. It is the *position* of lines, not measure, that determines all things. This is one of the propositions of the geometry of position that if any six points be taken

anywhere in the circumference of a circle and joined by consecutive straight lines in *any order whatever*, then will the three points of intersection of the three pairs of opposite lines lie in one and the same right line; an illustration of this is given in Fig. 1. The points are there taken in the order ABCDEF, and are joined by the six consecutive lines, AB, BC, CD, DE, EF, FA; the three pairs of opposite lines intersecting in three points, as shown by the dotted line. By opposite sides is meant every *first* and *fourth* line taken consecutively. Thus, beginning with the

Fig. 1.



line AB, the *fourth* line in consecutive order is DE, therefore AB and DE are opposite lines; so are BC and EF; so are FA and DC. Now, this proposition is true, not only for *any* six points taken *anywhere* on the circumference of a *circle*, and joined in *any order whatever* by consecutive lines, but for *any six points* on an *ellipse*, a *parabola*, an *hyperbola*, or, in fine, for *any* six points on *any* curve of the second order. The proposition is therefore general and comprehensive to the last degree. Yet it takes no account whatever of measure or metrical relations.

Two other propositions may perhaps afford a yet clearer idea of the character of the propositions of the *Geometry of Position*. It is, however, but just to state that they have not been *especially* selected for this purpose, but are here introduced mainly because they are to be applied later to the solution of problems in surveying. The first of these is to this effect: If A, B, C be *any* three points in *any* three concurrent lines, and A′, B′, C′ any three other points in the

same lines, and d, d', d'' the three intersections of the three pairs of connectors BC and B′C′, CA and C′A′, AB and A′B′, then are the points d, d', d'' in the same right line. The proposition is extremely general, the points may be taken anywhere in any three concurrent lines, and however great or small the angles which the lines make with each other, the proposition is invariably true. It is apparent from the figure that the statement of the proposition may be varied as follows; If any two right lined figures of three sides each, as ABC and A′B′C′ (Fig. 2), be so placed that the connectors of each pair of corresponding points A and A′, B and B′, C and C′ meet in a point P, then the three points d, d', d'' of intersection of the three pairs of corresponding sides BC and B′C′, AC and A′C′, BA and B′A′ are three points in a right line. No account is taken of the distance these figures are apart; no account is taken of the length of the sides nor of the angles they make with each other. Yet the proposition is in-

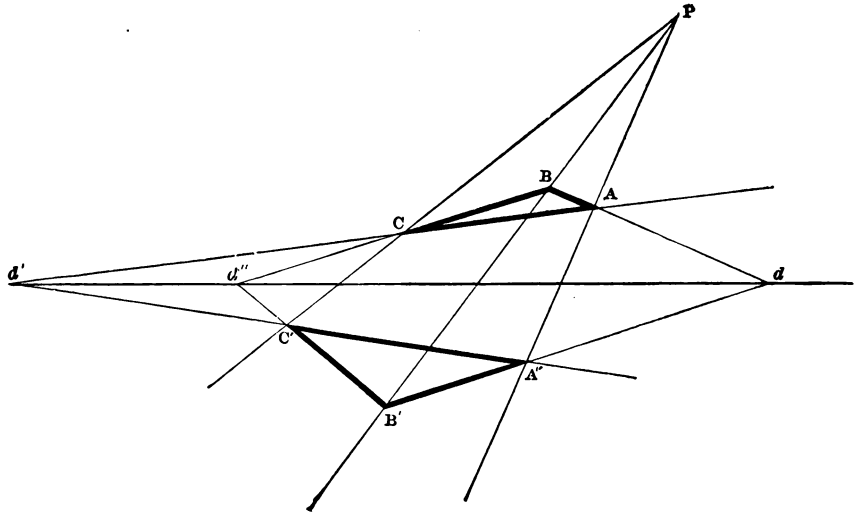


Fig. 2.

variably true for all sizes and shapes of figures having three right-line sides, when so *placed* that the lines joining each pair of corresponding points pass through a common point.

The second proposition is this: If A, B, C and D, Fig. 3, be any four points of intersection of *any* two pairs of concurrent lines and the points of concurrents *a* and *c* of these lines be joined by the line *ac*, then will the connectors AC and BD of the two pairs of opposite points A and C, B and D, meet the line *ac* in two points *b* and *d*, such that if either one with the two points *a* and *c* remain fixed, the other will also remain fixed for all possible positions of the points A, B, C and D. Or, to state it differently, if ABCD be any right-lined figure of four sides, and the two pairs of opposite sides AB and CD, BC and AD be produced till they meet, then will the two diagonals AC and BD, when produced, cut the line *ac* in two points *b* and *d*, such that if either one with the two *a* and *c* remain fixed, the other will also remain

fixed for all possible positions of the figure ABCD. This is illustrated by the figures on the right and left of ABCD. Here, again, no account is taken of measure. The length of the sides, the distance of the figure from the line *ab*, the measure of the angles has nothing to do with the proposition. So long as the *position* of the sides is such, that each pair of opposite sides intersect when produced, the proposition is *invariably true*.

III. ANHARMONIC RATIO.

This singular result should seem to indicate the existence of some fixed relation between the four points *a*, *b*, *c*, *d*, a relation which, on examination, is found to be of the very highest value in the application of the Geometry of Position to practical uses. The four points *a*, *b*, *c* and *d* will, it is evident, always lie in a right line, and possess relations and properties that may be briefly summed up as follows: The four points are four anharmonic points, constitute an anharmonic range, cut the line anharmonically,

have an anharmonic ratio, and, taken in pairs are the anharmonic conjugates of each other, that is to say, a and c are anharmonic conjugates, as are also b and d.

By anharmonic ratio is meant simply this: If any line, as ac, Fig. 3, be cut at any two points b and d, the line is cut into six non-adjacent segments, or segments having no end in common. These are ac and bd, ab and cd, and ad and bc. Now, if any two pairs of non-adjacent segments be taken, the ratio of ratios of these two pairs is the *anharmonic* ratio of the section of the line. Thus, if the sections taken be ac and bd, bc and ad, then the ratio of ratios is,

$$\frac{\frac{ca}{cb}}{\frac{da}{db}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Or again, to express it more simply, if a, b, c and d be any four points on a right line, and these points be taken in any order whatever, the ratio of the first and

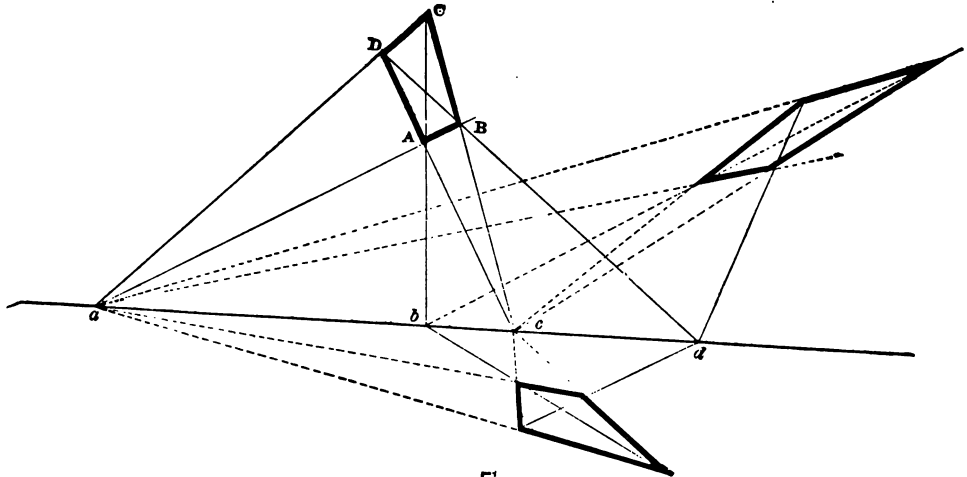


Fig. 3

second to the third divided by the ratio of the first and second to the fourth is the anharmonic ratio of the four points. So that the four points being taken in the order a, b, c, d, the anharmonic ratio is, by the above definition,

$$\frac{\frac{ac}{bc}}{\frac{ad}{bd}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

an equation precisely similar to equation (1). This ratio, for the sake of brevity, is written $[abcd]$, the order of the letters indicating the order in which the points are taken.

As these points may be taken in *any* order, and as the four letters representing them may be written in twenty-four different combinations, it should seem that there should be twenty-four different anharmonic ratios for each range of four points. When, however, the anharmonic ratio for each of these twenty-four combinations is written out, it appears that there are but six anharmonic ratios differ-

ing in value, and that each one of these six may be written in four different ways *without altering its value.* Thus taking the points in the order *abcd*, we have,

$$[abcd] = \frac{ca}{cb} : \frac{da}{db} = \frac{ca.db}{cb.da}$$

$$[badc] = \frac{bd}{ad} : \frac{bc}{ac} = \frac{bd.ac}{ad.bc}$$

$$[cdab] = \frac{ca}{da} : \frac{cb}{db} = \frac{ca.db}{da.cb}$$

$$[dcba] = \frac{db}{cb} : \frac{da}{ca} = \frac{db.ca}{db.da}$$

The anharmonic ratio $\frac{ca}{cb} : \frac{da}{db}$ may therefore be written in either of the four ways $[abcd]$, $[badc]$, $[cdab]$, or $[dcba]$. So may each of the five other ratios be written in four ways, without a change of value; the six different ratios and the four different ways of writing each, accounting for the twenty-four combinations of the points a, b, c, d.

When the value of the anharmonic ratio is unity, the points are said to be harmonic points, and the ratio an *harmonic ratio.*

In connection with this matter of anharmonic and harmonic ratio it is to be remarked, that the truth of this statement that the Geometry of Position takes no account of metrical relations, is not at all impugned by the fact that these *ratios* are based on the measure of the segments. They are, so to say, after-thoughts. The propositions of the Geometry of Position are wholly independent of anharmonic and harmonic ratios, which are, indeed, results derived from the Geometry of Position. In many instances, however, where a practical application of this Geometry is made, the necessity of obtaining the measure of lines arises, and then is it that these ratios are found invaluable. Such is the case when applied to the solution of problems in surveying, and on this account it has been judged best to briefly introduce the matter of anharmonic ratio before taking up the matter of surveying by the Geometry of Position.

IV. SURVEYING BY THE GEOMETRY OF POSITION.

We shall begin with the proposition illustrated in Fig. 3. The problems in Engineering Field-work, to the solution of which this proposition is applicable, fall under one of three classes.

1°. To prolong a line through or across some obstruction, as a house, a marsh or pond.

2°. To obtain the measure of an inaccessible or immeasurable distance.

3°. To locate and find the distance to a remote point.

1°. TO PROLONG A LINE THROUGH AN INVISIBLE POINT ON THE LINE.

The problems in this class are undoubtedly the most frequently met with of all, and the methods of solving them now in use, by perpendiculars, by equilateral triangles, by similar triangles, and by triangulation are so familiar that it is unnecessary to do more than recall

the fact that they all depend on angular and chain measurement, and that the character of the ground surrounding the obstacle to be passed will in general determine which of the three is to be used.

The Geometry of Position, on the other hand, affords two methods of passing round an obstruction, each of which is based on the proposition regarding the right line figure of four sides; is wholly independent of angular and chain measurement, and is applicable in *all* cases, whatever may be the nature of the obstruction or the physical character of the ground. Suppose, for illustration, that in locating a tangent on a railroad, represented by the line AX in Fig. 4, some obstruction as a house is met with at O. Now, by simply reversing the third proposition, which is illustrated in Fig. 3, it becomes applicable to the present case. For it is evident that if from any two points *a* and *b*, on a right line, two pairs of lines be drawn in such wise as to form by their intersection a figure ABCD, the

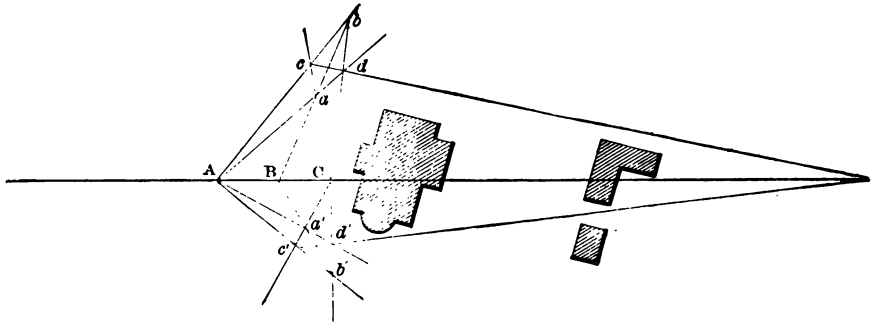


Fig. 4

two diagonals AC and DB will cut the line *ab* in two points *c* and *d*. To apply this to the solution of the problem in hand it is merely necessary so to arrange that the three points *a*, *c*, and *b* shall be on the line to be staked out and all three on the *same* side of the obstruction, while the point *d* is on the other side of the obstacle. The first condition is easily satisfied, since the points *a*, *b*, *c* may be taken *anywhere* on the line staked out before reaching the obstruction. All turns, therefore, on the determination of the point *d* at which the diagonal DB cuts *ab* produced, and this point may be found by either of two methods.

FIRST METHOD.

It will be remembered, in connection with the proposition, that if any three of the points remain fixed, the fourth point will also remain fixed for *all* positions of the figure ABCD as illustrated in Fig. 3. That is to say, if *a*, *b*, *c* are fixed points the diagonal DB will *always*, for all posi-

tions of ABCD, pass through the same fixed point *d*. In order to find *d* on the ground, it is necessary merely to lay out two figures, ABCD and A′B′C′D′, and find the intersection of the two diagonals CD and C′D′. This will be a point in *ab* produced. The simplest method of doing this is illustrated in Fig. 4, and is performed as follows. Select *any* three points, as A, B and C in the line previously located; set the instrument at B, and facing the obstacle turn the telescope off to the right of the line, and locate two points *a* and *b* in the line of sight such that from each of them a sight can be had past the obstruction. Now turn the telescope to the *left* of the line, and in the same way locate *a′* and *b′* in the line of sight. Move to A and sight to *b* and locate a part of A*b* near where it seems to cross a line joining *a*C. Then sight to *a* and locate a part of A*a* near where it seems to cross a line joining *b*C. Next sight to *b′* and *a′*, repeating the operation just described. This has all been done without moving from A.

Now set the instrument at C, sight to a, and locate c; also sight to b and locate d; then to a' and locate c' and to b' and locate d'. Finally, run out cd, locating a small portion of it about where it seems likely to cross ab produced, and the point in which c' d', when run out, intersects cd, in the point D in the line ab.

In passing round an obstruction by this method it is evident that no angles are read and no measurement of lines made. Nor is it necessary, for the angles which the lines from A, B and C, in the drawing, make with the line AB, has nothing whatever to do with the proposition, and hence the angles through which the instrument is turned from the line of sight is a matter not to be considered in the application of the proposition. Neither is the matter of the distance of the points a and b, a' and b', to be regarded, since the length of the lines in the figure has nothing to do with the solution. We are, therefore, in all respects at liberty to choose our ground and to turn the telescope to the right and left of the

line to be prolonged, without regard to the angle turned through. Two things, however, are to be avoided: If the points A and C are so chosen that AB = BC, then the lines cd and $c'd'$ will be parallel to to each other and to the line to be prolonged—in other words will not *intersect* AB. But inasmuch as these points A, B, and C are taken at pleasure and without measurement, it is quite improbable, in deed almost *impossible*, that AB will ever be made equal to BC, and hence no trouble need be borrowed on this account. Yet this serves to illustrate a feature of this system which will very often be found of use. When AB is exactly equal to BC the lines cd and $c'd'$ meet nowhere; consequently, when AB is a *little* greater than BC the lines cd and $c'd'$ will *meet*, but at a *great distance*. For instance, as proven in equation A′, if AC be one hundred feet long, and AB = 50.003 feet, BC will equal 49.997 feet, and CD will be over *one hundred and fifty-seven miles* long. Had CB been 49 feet, then CD would have been 2450 feet. To apply

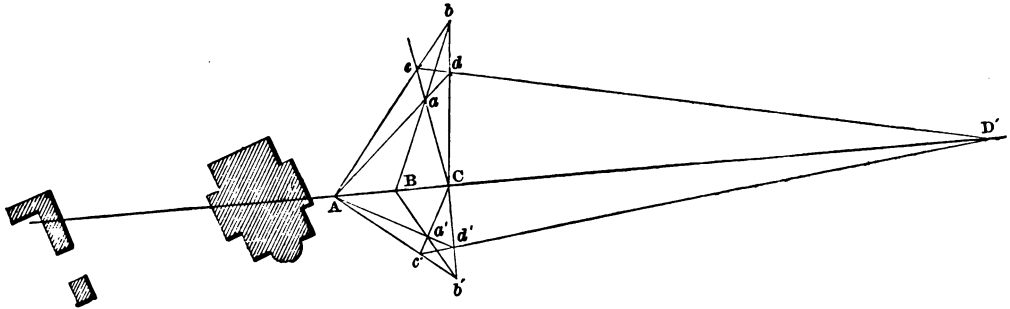


Fig. 5

this: if at any time it is necessary to prolong the line through a *series of obstacles*, as illustrated in the figure 4, the points A and C have only to be taken so that BA, judging by the eye, is a few feet greater than BC, and the lines cd and $c'd'$ will meet beyond the obstacles. Finally, that the lines cd and $c'd'$ shall *invariably* meet *beyond* the obstruction, the lines of sight from B to b and b' must point towards the obstacle. The consequence of pointing these backwards is shown in Fig. 5. The diagonals cd and $c'd'$ will, even in this case, meet AB, but at a point behind A, as at D′.

THE SECOND METHOD.

The second method of determining the point D is much shorter than that just given, and not less accurate. Take *any* point as a (Fig. 6), not in the line to be prolonged, from which it is possible to see past the obstruction on each side. Set the instrument at this point, and turning the telescope in the direction of D (so as

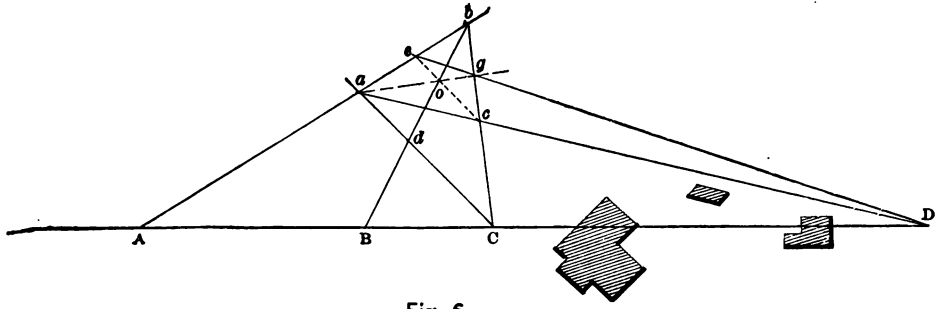


Fig. 6

to clear the house or whatever the obstacle may be), locate any point in the line of sight, as *c*, and also locate on the ground a small portion of the line near where, in all probability, it will cut AC produced. Then (from the same point *a*) sight to *any* point in the line AC, to be produced, and near the obstruction at C, staking out a small part of *a*C as *ad*. Next sight (from *a*) to *any other* point in the line AC further from the obstacle than C, locating a portion of this line *a*A as *ab*. The location of the lines *ad* and *ab* may be speedily effected in this wise. The points C and A are taken *anywhere* in AC, as is also the point *c* in *a*D. To locate *ad*, therefore drive a few stakes about where the line of sight *ac* seems likely to cross *c*A. To locate *ab* find a few points in the line *a*A which seem to be in range with *c* and C.

After determining these lines on the ground, turn the telescope so that the line of sight cuts C*b anywhere* between *c* and *b*, and locate the line *ag*. Now move the instrument to *c*, sight to C, and find *g*

and *b* exactly; then sight to A, and find *d* exactly. Move to *d*, sight to *b*, and find *o*. Return to *c*, sight to *o*, and find *e*. Finally move to *e*, sight to *g*, and find the intersection of the line *eg* with the line *ca*. This will be the point D, a point in the line AC produced through the obstruction.

If these two methods of prolonging a line through an obstruction, as illustrated in Figures 4 and 6, appear quite complicated, it should be remembered that the lines of the figure have no existence on the ground, while a few applications to practice will show the utility of the system, and the ease and rapidity with which the work may be done. They are superior to the old method, in that they are wholly independent of both angular and chain measurements, and thus leaving the transit man free to choose his ground are applicable in any case and on all kinds of ground. As to their accuracy, some idea may be formed by performing on paper with a ruler and pencil the operations described in the text. Repeated

trials have demonstrated their accuracy in the field.

These and indeed any other methods of passing around obstructions, would not be of much practical value did they fail to afford a way of determining the break in the line prolonged; in other words CD, (Fig.6) a problem the Geometry of Position solves in a singularly beautiful way. Under the head of anharmonic ratio it has been stated that the four points *a*, *b*, *c* and *d* of Figure 3 are four anharmonic points, and taken in any order whatever afford an anharmonic ratio. Taking them therefore in the order A, C, B, D, (Fig. 4) the anharmonic ratio is

$$AB : CB :: AD : CD *$$

solving this we have

$$\frac{AB}{BC} = \frac{AD}{CD} = \frac{AC+CD}{AD-AC}$$

$$\therefore AB \times AD - AB \times AC = BC \times AC + BC \times CD.$$

* This is evidently the harmonic ratio of the points when taken in the order ABCD.

But $AD = AC + CD$

$\therefore AB \times AC + AB \times CD - AB \times AC$

$= BC \times AC + BC \times CD.$

reducing $CD(AB - BC) = BC \times AC$

$$CD = \frac{BC \times AC}{AB - BC} \quad . \quad . \quad . \quad (A)$$

To obtain the valve of CD, it is necessary to know the values of AB and BC.

By assuming different values for these segments and substituting in equation A, two singular results follow. Suppose $AB = 40$ and $BC = 20$ feet; then

$$CD = \frac{20' \times 60'}{40' - 20'} = 60'$$

That is, if the distance BC be equal to one half the distance AB, then CD is equal to AC, a fact worth remembering when the first method of prolonging the line is used, as in that case the points A, B and C are taken at pleasure, and can therefore be so taken that $BC = \frac{1}{2}AB$. Again, if B is taken midway between A and C, so that AB and BC have each the same value, say 30 feet, then

$$CD = \frac{30' \times 60'}{30' - 30'} = \infty$$

or the point D is at an infinite distance from C and the lines cd and $c'd'$ (Fig. 4) are parallel to each other, and to the line AC; a demonstration of the statement made regarding the first method of prolonging a line, namely, that in selecting the points A, B and C on the line AC, they must be so taken that B *shall* not be midway between A and C.

Finally, if AB be less than the segment BC, as, for example, $AB = 10'$, and $BC = 15'$, then

$$CD = \frac{15' \times 25'}{-5} = -75'$$

or the point D will fall $75'$ to the left of C, or on the *same side of the obstruction* as the points A, B and C, as illustrated in Fig. 5. In practice, the second and third results are never likely to occur. For, if the line be prolonged by the second method, neither *can* occur, since *ac* is *so taken* as to pass the obstruction, the point B, the conjugate of D, *cannot possibly* fall so that AB shall be equal to or less than BC. When the first method is used, the

point A, B and B are taken at pleasure, and may, therefore, be so taken as to avoid all trouble.

The accuracy of Eq. A may be tested graphically by assuming different values for AB and BC, laying them off to a scale on a right line, drawing from A *any* two lines, as A*b* and A*c*, of indefinite length from B *any one* line, cutting those from A in any two points, as *d* and *b*; from C, two lines through *d* and *b*, cutting the two from A at *a* and *c* respectively, and drawing *ac* till it meets AC produced in D. Then, measuring CD by the scale used to lay off AB and BC, the value obtained for CD should agree with that found by substituting the values assumed for AB and BC in Equation A.

The proposition of the Geometry of Position relative to the four points at the intersection of two pairs of concurrent lines, (Fig. 3) is readily applicable to the solution of a second class of problems of very common occurrence in surveying—the measurement of inaccessible distances.

The cases which, in practice, fall under this head are almost innumerable; as the measurement of the width of a river, the distance across a marsh, the distance of a lighthouse or a beacon from the shore, &c. Yet the solution of any one of them by the Geometry of Position is ample enough to cover them all. To take the first example above cited, let it be required to find the distance of a point C (Fig. 7) on one bank of a river, from a point D, on the opposite bank.

To accomplish this, set the instrument at the point C, sight to D, reverse the telescope, and take any point in the line of sight as A. The line CA will then evidently be a continuation of the line CD, the distance to be measured. Now, take *any* point *off* the line CA as *a*, such that both C and D are visible from it, set the instrnment at this point, sight to D, and take *any* point in the line of sight *a*D as *c*. Then sight to A, previously chosen anywhere on CA, and "stake out" that part of the line *a*A, which seems to cross a line joining *c*C. Sight next to C (the

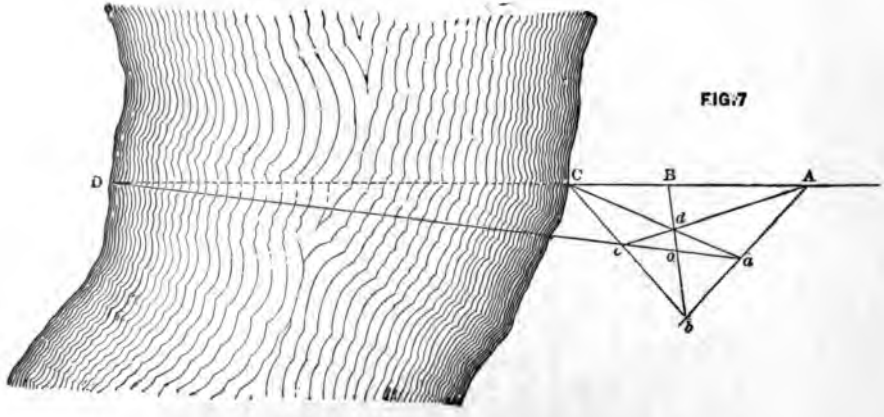


FIG.7

instrument being still at a) and "stake out" that portion of the line aC which seems to cross a line joining c and A. Move now to c, sight to C, and locate b; then sight to A and locate d. Set the instrument at d, sight to b, and find B where the line of sight cuts the line CA. Finally, measuring the distances CA and BA and substituting in equation A.

$$CD = \frac{BC \times AC}{AB - BC'}$$

the distance across the river is found. Thus if BC = 48.99 feet, then BA = 51.01 feet, and

$$CD = \frac{48.99' \times 100'}{2.02'} = 2{,}425.24 \text{ feet.}$$

It is not, however, to be supposed that this proposition of the Geometry of Position does not equally apply to cases in which it is desirable to find the distance and direction of two or more objects from some fixed point. To take an example, suppose at A and C, in Figure 8, are two objects whose distances and directions from some fixed point b it is desirable to know.

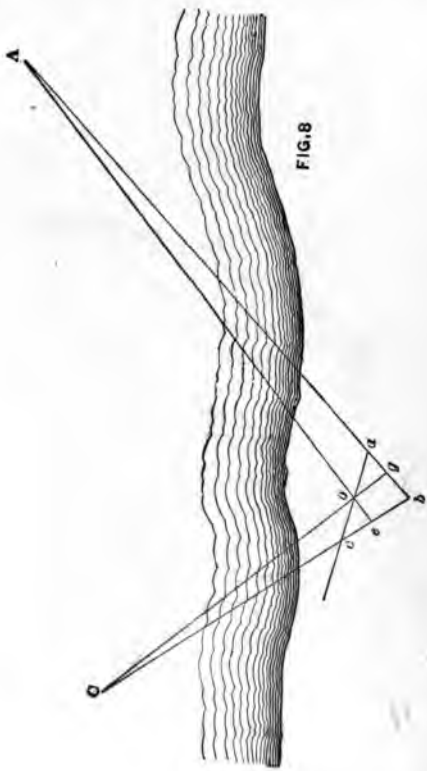


FIG.8

Referring to Fig. 7, it is evident that if *a*, A, *d*, *o* be considered as the four points of intersection of two pairs of diverging rays from *c* and *b*, and *bc* the line joining the points of divergence (*b* and *c*) of the two pairs of rays, then will the diagonals *ad* and *o*A (Fig. 8) of the figure *a*A*od*, intersect the line *bc* in the two points *c* and *e* respectively. The four points *b*, *e*, *c*, C, will then form an anharmonic range, and from the principles of anharmonic section, already explained, it results that

$$cC = \frac{ce \times cb}{cb - ce}$$

Again, regarding the points *b* and *a* as the points of divergence of the pairs of rays intersecting each other in the four points C*cod*, and the line *ba* as the connector of the points of divergence *b* and *a*, the two diagonals *cd* and C*o*, of the figure C*cod* will intersect the connector *ab* in the two points A and *g* (Fig. 8) respectively. Those two points will form with *a* and *b* an anharmonic range from which may be obtained

$$aA = \frac{ga \times ab}{bg - ga}.$$

But $bC = bc + cC$ and $bA = ba + aA$, from which the distance required may be readily obtained.

To apply this to the case just taken. The point b having been selected, the solution of the question depends on finding the distances bC and bA and the bearings of these lines. To obtain these, set the instrument at *any point* between b and the objects, as o, and sighting in *any direction* such that the line of sight *does not cut* the line CA, anywhere between C and A, locate a few points on the line of sight about where it seems to cut a line joining b and C. Reverse the telescope, and in a similar way locate a few points about where the line of sight seems to cut a line joining b and A. Sight next to C and locate g as nearly as possible; and then sight to A and locate e approximately by setting a few stakes where the line of sight Ao seems to cut bC. This done, move the instrument to b, sight to C, and locate e and c with great exactness

Sight next to A, and locate *g* and *a*. Now, having measured *be* and *ec*, as also *bg* and *ga*, and substituting these values in the two equations above given, the values of *c*C and *a*A are found; which, added to *bc* and *ba*, give the required distances *b*C and *b*A. The magnetic bearings are, of course, taken when the instrument is at *b*.

Without stopping to name over the many instances in which it will be advantageous to apply the two methods of measuring distances illustrated in Figs. 7 and 8, I shall pass to the consideration of a more important matter—the degree of accuracy of the two methods.

Beginning with the first mentioned method (that illustrated in Fig. 7), it is needless to observe that the point A on the line *c*D may be taken at any distance from C. To simplify the measurements, therefore, the distance AC may be taken at one hundred feet, and laid off at once by means of a tape. But the distances AB and BC must be measured with great care, and to smallest fraction of

a foot. For it is evident from equation A, that as the value of BC approaches that of AB, the value of CD in the expression

$$cD = \frac{Bc \times Ae}{BA - Be}$$

approach infinity, because when $AB = Bc$ the values of cD *is* infinity. It follows, therefore, that when the distance to be measured is quite a long one, the difference between AB and BC will be very small, indeed but a decimal of a foot. To illustrate with an extreme case, suppose the distance to be measured is 499,950 feet, or something over 94.7 miles, then

$$cD = \frac{49{,}955' \times 100'}{50{,}005 - 49.995} = 499950 \text{ feet.}$$

A difference, then, of .01 of a foot between AB and BC will correspond to a distance $CD = 94.74$ miles. To take a more likely case, suppose $AB = 50.1'$, and $BC = 49.9'$ feet, the distance AC being one hundred feet, then

$$CD = \frac{49.9' \times 100'}{50.1' \times 49.9} = 24950 \text{ feet.}$$

a distance equal to 4.725 miles. In proportion as the distances to be measured are shorter, the differences between AB and BC are larger. Thus, a distance of 2425.24 feet corresponds to a difference between AB and BC of 2.02 feet, while a distance of 1199.37 feet will correspond to a difference between AB and BC of 4.002 feet. A distance of 1 foot on CD will therefore have an exceedingly small difference. If the distance CD be very great, as five thousand or six thousand feet, the difference between AB and BC corresponding to a foot on CD will be at least some ten thousandths of a foot; if the distance CD be, on the other hand, from one to two thousand feet, the difference between AB and BC corresponding to a foot on CD will be about one thousandth of a foot. To obtain accurate results it thus becomes quite necessary to be able to measure the distance BC to the ten thousandth of a foot for very large distances, and to the thousandth of a foot for all small or ordinary distances. Thus, a distance of 1199.37 feet corre-

sponds to a difference between AB and BC of 4.002 feet, but an increase in the difference to 4.004 of a foot will correspond to a value for CD of 1198.76 feet. Here, therefore, a distance of .61 of a foot is measured by a difference of .002 of a foot.

This fineness and accuracy of measurement constitutes, perhaps, the main objection to the methods discussed above. Yet it is one not impossible to overcome. The distance AC is, for instance, one hundred feet, measured off with all possible accuracy, the temperature and horizontal position of the tape being, of course full considered. Now the point B can never fall anywhere on AC, except between the middle point of AC and C. For if it falls *exactly* midway between A and C, the point D is at an infinite distance from C, or CD is infinitely great. Neither can B fall between A and the middle point of AC, as in that case the point D would fall on the opposite side of A, or, in other words, the point A would be *between* D and B. This never

can happen in either of the two methods given above, because A and C are chosen at pleasure, and the point D being *sighted to first*, its anharmonic conjugate B must invariably fall between A and C. As a consequence of this fact, it follows that the measurements to the hundredths and thousandths of a foot need not begin until the fiftieth foot has been passed. Nor, on the other hand, is it necessary that the measurement shall extend for any great distance. If AB be equal to 50.003, and BC 49.997 feet, the value of CD will be over one hundred and fifty-seven miles; if, on the other hand, AB be 55 feet, and BC 45, the difference will be ten feet, and CD will be 450 feet. It will never be necessary, therefore, to go so near the fifty-foot point as .01 of a foot, nor so far away as ten feet. The fine measurements, in other words, will be confined to ten feet, and may be obtained in a number of ways that readily suggest themselves. The simplest is by means of a well-constructed levelling rod, with a sliding target. If this be used the center

point of the distance AC should first be carefully found, and one end of the rod placed exactly over the point by means of a plum bob, and the rod put horizontally in line with the instrument. It is best to have some simple support for the rod to keep it off the ground, and to enable it to be placed truly horizontal by the aid of a bubble. This done, and the sight *bd* (Fig. 7) taken to determine the point B, the target may be moved along till it crosses the line of sight, and the distance AB obtained to the thousandth of a foot. If the distance to be measured is very large the vernier must read to ten thousandths of a foot, or the results obtained will be utterly worthless.

The extreme accuracy and fineness of the measurement, thus necessary when long distances are to be measured render it doubtful, to say the least, whether the methods in question are superior or more practicable than those now in use. The accuracy, however, to be exercised in the measurement of the one hundred feet required between C and A (Fig. 7) is no

greater than should be exercised in the measurement of ordinary lines in city surveying, while distances commonly met with in surveying, as the width of a river or stream, the distance over a marsh, etc., can be obtained without measuring finer than the one thousandth of a foot, which may be done with a common levelling rod. For example, if the distance AB = 51.16 feet, and AC = 100′, then the distance BC = 48.24 feet, and Eq. A,

$$CD = \frac{48.24' \times 100'}{3.52'} = 1370.45.$$

But if AB = 51.761 feet, then BC = 48,239 feet and

$$CD = \frac{48.239' \times 100'}{3.52'} = 1369.647$$

a difference of about eight-tenths of a foot. The question, then, as to whether this method is better than the old method by logarithms, resolves itself into this: is it better to make one short accurate measurement on the ground, or to make large linear and angular measurements on the ground, and solve by the rules of trigonometry?

For the solution of such problems in surveying as do not require the measurement of distances, the Geometry of Position affords methods, the merits of which are unquestionable. Such problems are those requiring the location of lines and very likely to occur in laying out and dividing land, as also in chain surveying and in town surveying. To take an instance, having two converging lines given as AB and CD, Fig. 9, let it be required to pass a line through their invisible point of intersection. If it be merely required that the line shall pass through the point of intersection, the problem is of the most general form and may be solved as follows: Set the instrument at *any* point not within the lines, as a, and sighting across them both locate the points b and b' where the line of sight cuts them. Then turn the telescope so as to cut them at any other place and locate the points c and c'. Turn the telescope through another angle and locate the points d and d' as before. Move then to b, and sighting to c' locate

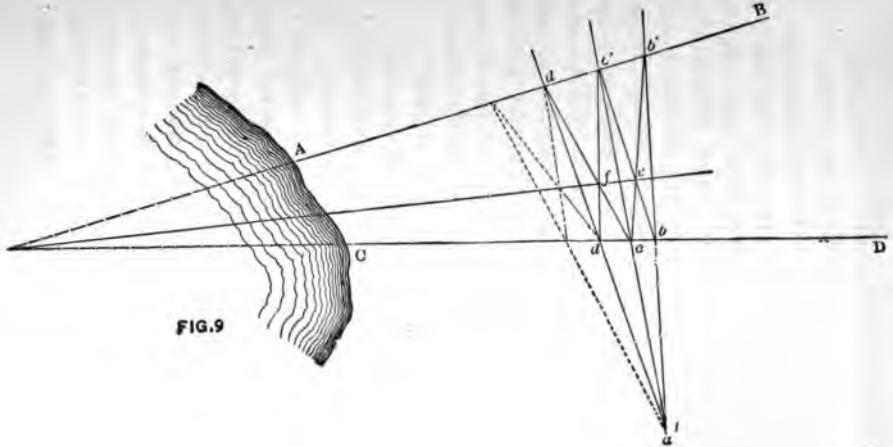


FIG.9

the middle part of the diagonal bc'. Then move to c, sight to b', and find e exactly, and continue this operation till the diagonals cd' and dc' are located and f found. The line joining ef passes through the point of intersection of AB and CD. Although two points are enough to determine the line ef, an additional point may be obtained as shown by the dotted lines in the figure. In using this method it does not make any difference where a is taken without the line, nor is any account taken of the angles the lines ab, ac, ad, etc., make with each other.

If the problem takes a more limited form, and the line must pass through a *given point* and the intersection of AB and BC, the solution may be effected in this wise. Let P (Fig. 10), be the given point. Set the instrument, as in the last case, at any point not between the lines as a, and locate, as before, the points b and b'; c and c'; d and d'. Now move to the point P through which the line is to pass, and sighting to b', locate c'' where the line of sight cuts ac. Also sight to b

and determine c'''. Move to c'', sight to d', and stake out a portion of the line c'' d', beyond d'. Finally, move to c''', sight to d, and find e exactly. The line through Pe will pass through the intersection of AB and CD. If the lines AB and CD are very far apart, some time and trouble may be saved by beginning at b, sighting to P, and find c'''. Then setting the instrument at c''', sighting to d, and locating a part of the line c''' d. Repeating this on the other side, beginning with b' the point e is readily found.

When the point P, instead of being *within* the angle made by AB and BC, as to the case in Fig. 10, lies *without* the angle, as shown in Fig. 11, the line may still be found as in the previous case.

Where it is possible to put the instrument (when used) exactly on the line, a simple method is to begin at b (Fig. 11), sight in any direction and determine b', and then a, anywhere on bb' produced. Then sight to P, and find c''' approximately. Next take any other point as c, sight to a, and find c''' exactly, and c''

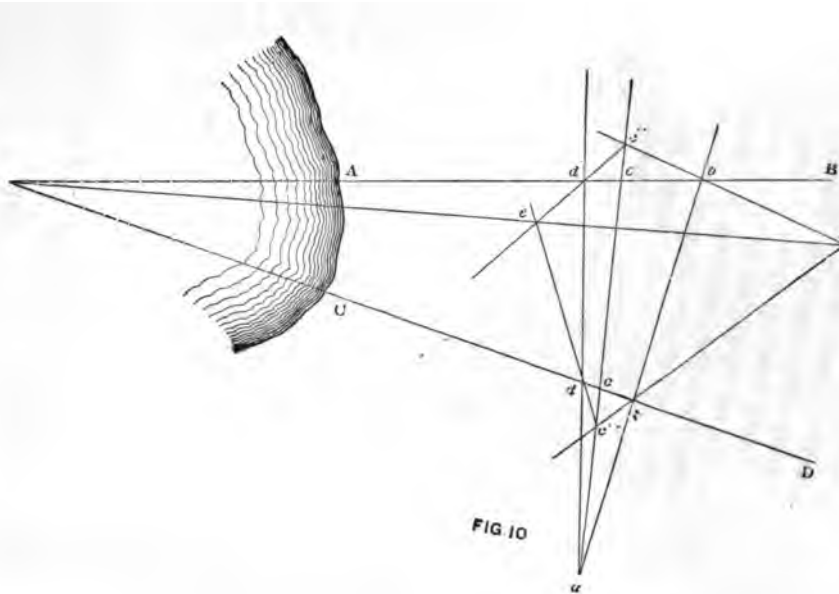


FIG. 10

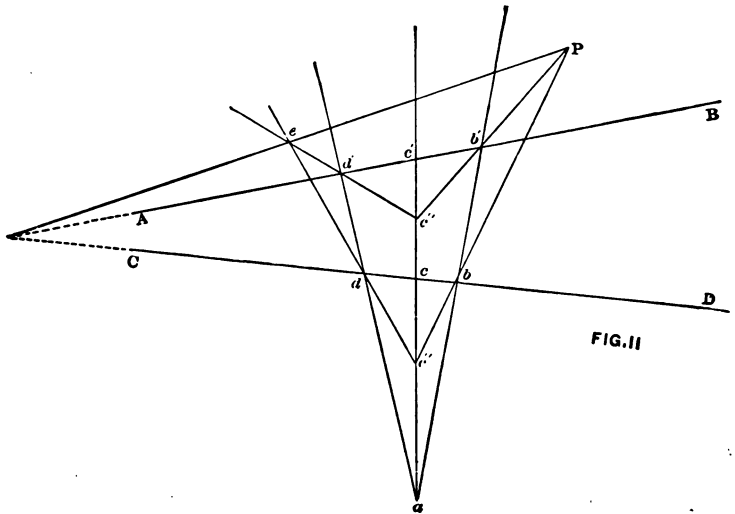


FIG. II

approximately. Then take d anywhere on the line CD, sight to a, and find d', then to c''', and find e as nearly as may be, then find c'' by moving to b', and finally e' by moving to c'', and sighting to d'.

Of the two, the first method is perhaps of more general application, as it affords simple methods of bisecting, trisecting, &c., the angle formed by the two lines, as will be shown when considering the application of the Geometry of Position to drawing. It likewise affords a solution to the problem—of passing a line through a given point parallel to a given line. Let AB (Fig. 12) be the given line, and P the point through which it is required to pass a line parallel to AB. Referring to Fig. 9, it will readily be seen, that, if C′ and d' be regarded as the points of divergence of two pairs of rays intersecting at $fdac$, then will dc be one diagonal and af the other. But if this latter, af, be made to cut $d'c'$ just midway, then will cd be parallel to AB. This gives the solution for the problem in

question. Lay off on the given line anywhere, two equal distances *bc* and *cd*. Through the point *b* and the given point P draw a line, and in it take *any* point as *a*. From *a* draw lines of indefinite length through *c* and *d*; join *dP* and mark the point *c'* in which it cuts the line *ac*. Through *b* and *c'* draw a line till it cuts *ad* at *d'*. Then is *d'* a point on a line through P parallel to A B.

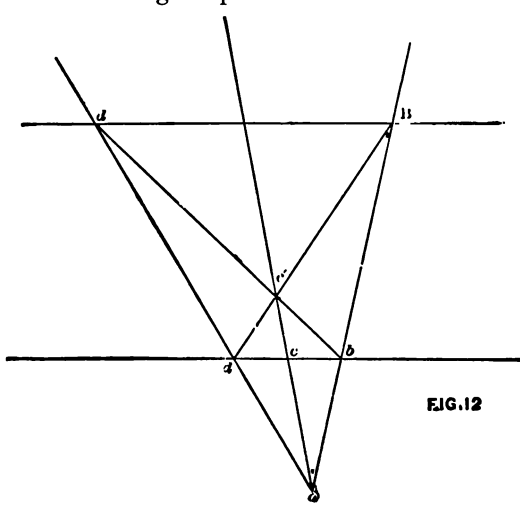


FIG. 12

The solution is undoubtedly of most use in chain surveying; yet it is evident that as affording a means of obtaining the bearing of an inaccessible line, of which two points only are to be seen, it is of value in all branches of surveying. By obtaining a line *parallel* to the inaccessible line, and then finding its bearing the desired result is accomplished.

CO-ORDINATE SURVEYING.

BY

HENRY F. WALLING, C. E.

CO-ORDINATE SURVEYING.

RELATION OF SURVEYING TO GEODESY.—Surveying is commonly defined as the art of measuring, laying out and dividing land. The similar but more comprehensive art of geodesy applies to the earth itself, as indicated by the etymology of the word, which is derived from *γη, the earth,* and *δαιω, I divide.* Its objects are, the determination of the form and dimensions of the earth and of its different portions; the continents and islands, with their lakes, rivers, mountains, civil divisions, etc. Its processes, like those of astronomy, upon which, indeed, it is to a considerable extent dependent, require instruments of a high degree of precision and mathematical computations of great refinement.

GENERAL GEODETIC CO-ORDINATE SYSTEM.—In the operations of the great

geodetic surveys of the world, including for our own country those of the United States Coast Survey, positions are finally determined by referring them to co-ordinate planes. The plane of the earth's equator forms one of these co-ordinate planes, and that of an assumed standard meridian another. If we should assume that the earth is spherical in form, with continual elevations, the position of any geodetical point could be fixed by determining the direction in space of its radius vector, as referred to the two co-ordinate planes, and the length of this radius vector, the origin being at the center of the earth. The angle with the equatorial plane made on either side by the radius vector would be the latitude; the angle made by its projection upon the equatorial plane with the meridianal plane measured in either direction around a semicircle would be the longitude, and the length of the radius vector, or rather its excess over that to the level of the sea, would be the altitude of the place determined. Owing, however, to the spheroidal form

of the earth, latitudes as observed and established do not exactly represent the co-ordinate angles here deccribed. The actual latitude of a place is the angle made by a normal to the earth's surface at that point with the equatorial plane. The geocentric angle could easily be computed from the latitude if the earth were an ellipsoid of revolution of known eccentricity.

IRREGULAR FORM OF THE EARTH.—But it has been found that this is not the true form of the earth. The equator, and probably all parallels of latitude taken at the level of the sea, vary more or less from true circles, indicating a want of homogeneity in the materials which make up the earth's volume. Longitudes are measured by noting the earth's rotation angles on the undoubted assumption that its angular velocity is strictly uniform. By noting the difference in time between the passage of a normal to the earth's surface at any particular point, and of another normal at any point on the standard meridian, across a celestial meridian,

we obtain an angle which is called the longitude of the former point, the longitude of the standard meridian being zero. Irregularities in the form of the earth of course change the direction of normals to its surface, and the co-ordinate system of latitudes and longitudes, used in geodesy and navigation, is correspondingly irregular.

ITS DETERMINATION A DIFFICULT PROBLEM.—In consequence of these irregularities, the problem of determining the exact form of the earth is an exceedingly difficult one. Instruments of the highest degree of precision must be used by skilled observers, and the combined results of a vast number of careful observations over widely extended areas must be subjected to the most refined mathematical investigation before its full solution can even be approximately obtained.

PROGRESS MADE IN ITS SOLUTION.—Its investigation has been going on, however, for one or two centuries, and very fair progress has been made. The great national surveys of the world have been

conducted by men eminently qualified for the task, and the results of these surveys, so far as completed, seem to approach quite near to the attainable limits of accuracy.

UNITED STATES COAST SURVEY.—This is particularly the case with our own Coast Survey, which is probably unsurpassed, if not unequaled in precision and general accuracy. Our country, however, has thus far failed to realize some very important advantages which might be derived from these Coast Survey operations although they have been sufficiently carried out to render them available over considerable portions of the country.

OBJECTS OF THIS ESSAY.—It is the object of this paper to point out a simple method by which the high degree of precision which accompanies the Coast Survey work may be made available, in the ordinary operations of land surveyors and civil engineers in those districts over which the Coast Survey triangulations have been carried, and at the same time to call attention to the importance of an

extension of these triangulations over the entire country, either by the general or by State governments. One of the disadvantages of our peculiar confederate form of popular government is an apparent inability or indisposition to undertake works of acknowledged and eminent utility, unless they are popular with the masses or with those who control the machinery of elections. It is certainly the experience of foreign countries, including several less wealthy and prosperous than our own; that these surveys are many times more valuable than their cost, in the aid they afford to the carrying out of internal improvements, to the equalization of taxes, and to many of the necessary operations of general and municipal governments, as well as of private individuals. But in this country where legislation usually follows, instead of leading, the expressions of general public opinion, such works are likely to await the slow and gradual cultivation of the popular mind to a proper appreciation of their great utility.

CONGRESSIONAL LEGISLATION.—In the meantime, it must be admitted that Congress has enacted a very liberal and wise law, by which the Coast Survey is authorized to extend its triangulation over any State in which scientific surveys have been provided for by the State Legislature. Moreover, the Superintendent of the Coast Survey evinces a disposition to construe this act with great liberality. It is understood that he will, where thorough topographical surveys are undertaken by State authority, cause the subsidiary triangulation to be carried to any reasonable degree of minuteness, with the same refined accuracy which has characterized the work already done by the officers of the survey. Indeed, with their elaborate determination of local irregularities in the form of the earth, and their well-organized corps of skilled observers and computers, they have an immense advantage, which could hardly be soon attained by any new organization, for extending the triangulation over whole

States and carrying it to the secondary and tertiary stages.

TRIANGULATION OF MASSACHUSETTS.—The State of Massachusetts is entitled to the credit of being the first of the United States to recognize the importance of having her territory carefully surveyed and to commence upon such a work. More than forty years ago a triangulation was commenced which was completed in 1842. This triangulation will compare favorably with the Coast Survey work and with the other geodetic surveys of the world. But a surprising indifference to its value, or potential utility, seems to have prevailed, and up to the present time no further use has been made of it than to adopt it for the basis of such State and county maps as have from time to time been published. These maps give only the horizontal locations of objects obtained from such imperfect surveys as could be paid for by the sale of maps, published entirely by private enterprise, no assistance being given by the State.

STATE SCIENTIFIC SURVEYS.—At the present time, the subject of scientific surveys is being agitated in the States of Massachussetts, Rhode Island, Connecticut and New York. The latter State appropriated $20,000, in 1876, for preliminary organization, to effect which it appointed a skilled director. The other States mentioned have organized commissions to investigate and report on the subject.

REFORM NEEDED IN LAND SURVEYING.—Before proceeding to describe the proposed combination of surveying with geodesy, a brief consideration may be permitted of the urgent need of reform in the prevalent methods of land surveying and of writing descriptions in conveyances of land, based, as many of these descriptions necessarily are, upon imperfect surveys and frequently upon no surveys at all. Lawyers and land surveyors are perhaps most familiar with the shortcomings of these conveyances in failing properly to describe the property conveyed. A large share of the litigation of the country arises from

this cause. Indeed, the laxity in this respect is something almost incredible. Scarcely one deed of conveyance in a hundred will be found to contain such a description of the land conveyed as would fix its location with certainty, if the fences, walls or other inclosures should become obliterated, a contingency which is quite likely to arise.

CONFLAGRATION OF DETROIT.—An occurrence of this kind on a rather extensive scale took place in 1805, when the city of Detroit was devastated by fire, and so thoroughly destroyed, that it was found quite impossible to ascertain the former boundaries of estates. The restoration was entrusted to the Governer of the State and a council of judges, who could find no better way out of the difficulty than by re-laying out the city on an entirely new plan, dividing the lots among the former owners as equitably as possible.

Notwithstanding this experience, the lines of streets and lots in Detroit are now so uncertain that disputes and litiga-

tion in regard to them are of continual occurrence. The same is true in most of the cities and large towns of the United States, especially in suburban districts and growing villages where land is rapidly increasing in value.

FAULTY DESCRIPTIONS IN LAND CONVEYANCES.—If we examine the descriptions given in land conveyances, we shall find that they usually fail to fix either the location of the property by references to permanent land marks, or even the position of its boundary lines relative to each other. Frequently the tract conveyed will be bounded in the deed by the several tracts adjoining, of which the only description given is to state the names of the supposed owners. In many deeds all dimensions are omitted, and only an indefinite approximation to the quantity of land conveyed is given, the statement being that it contains about so many acres, "be the same more or less." Where good permanent division fences, walls, hedges, ditches, streams, shore lines, bound-stones, stakes, &c.,

mark the boundaries, and are properly described, such descriptions may answer the purpose so long as the boundaries remain unchanged, although such indefiniteness as to quantity would hardly be tolerated in the sale of other kinds of property.

OBLITERATION OF MONUMENTS.—But physical monuments are continually becoming obliterated even when well defined at first. It is said to be an old custom in some parts of the country to take children once in every year to important boundary corners, where monuments have been erected, to the location and surroundings of which the careful attention of the children is directed. If on a subsequent visit their memory is found to be at fault, it is refreshed and deepened by combining with it that of a sound flogging.

UNRELIABILITY OF SURVEYS. — Even where surveys have been made, they are, in many cases, so unreliable that the recollection of old residents in the vicinity, considerately stimulated, perhaps, in

their youth, in the way described, is more reliable in determining the proper location of lost boundaries, than the retracing of old surveys. This is not surprising when the modes of surveying and the character of the instruments used are taken into consideration.

Outside of cities and larger towns, the instruments usually employed in surveying are the chain and compass. The method is to perambulate the boundary line of the tract to be surveyed, taking the magnetic bearing with the compass, and measuring with the chain the lengths of each side of the polygon forming the boundary.

IMPERFECTION OF THE COMPASS.—Now, provided the bearings thus taken were precisely measured angles from fixed parallel meridians, whose directions could always be easily ascertained, when a re-survey should be needed, no better method of noting directions could be desired. But this is far from being true. The magnetic force, to which the direction assumed by the needle is due, is quite irregular in its actions, changing

its direction continually, backwards and forwards, even during the different hours of the day, while larger oscillations extending sometimes to many degrees of arc take place in irregular cycles, perhaps one or two centuries in duration. And this is not all. In regions containing metallic deposits, especially magnetic iron ore, very irregular and powerful local disturbances of the magnetic force arise, causing the needle to take widely different directions, even at points in close proximity to each other, thus destroying parallelism of action, and rendering the compass quite useless for ascertaining true directions.

Beside the uncertainty of the magnetic meridian, there is an incapacity of precision in the use of the compass for measuring angles. The needle must swing clear of the graduated limb, and cannot be suspended for field use with very great delicacy. In practice it is usually impossible to read a magnetic bearing with greater precision than to the nearest ten minutes of arc.

THEODOLITE AND SURVEYOR'S TRANSIT.—The theodolite and the surveyor's Transit are instruments of far greater precision, and are generally used in cities, and for more valuable farm lands, also for engineering works, roads, railroads, &c. Those in ordinary use measure angles to single minutes.

In land surveying, the common method of using these instruments is to measure the angles formed by the sides of the bounding polygon with each other, and sometimes for verification with one or more diagonals. The compass needle, usually attached, affords an approximate means of ascertaining azimuths, but with no more precision than with the ordinary compass. It is, accordingly, quite as difficult to retrace obliterated boundaries with these instruments as with the compass, unless one or more well-defined lines remain for reference.

SOLAR COMPASS.—Burt's solar compass, now used in the surveys of the public lands at the West, for running out the parallel and range lines, is a great im-

provement upon the magnetic compass in the accuracy of its azimuths if not in the precision of reading minute angles. The direction of the sun, with proper adjustment of the instrument for the latitude of the place, and declination of the sun, and hour of the day, affords, of course, a reliable means of obtaining the true azimuth of an observed line with as much precision as the mechanical construction of the instrument permits. The use of the solar compass, however, is limited to sunshining or slightly cloudy days, the middle portions of which, moreover, are unfavorable to accurate observations, and at best the precision of its angular measurements is much inferior to that attainable with the transit. Nevertheless, for the preliminary surveys of wild lands, where no trigonometrical survey has been made, and where rapidity and economy are required, as in the government surveys of western lands, it is a very convenient and useful instrument.

MEASUREMENT OF DISTANCES. — The

direct measurement of distances is attended with even more difficulty than that of the determination of directions. It is accomplished by the repeated application of the chain, tape or measuring rod, as nearly as possible, upon the line to be measured. Passing over inaccuracies in the length of the measuring standard, arising from imperfections of construction, inequalities of temperature, changes of length by stretching, kinking, &c., the difficulties of making the direct applications are frequently quite serious. It often happens that access to all parts of the line to be measured is difficult, if not impossible. In fact, the boundaries of improved properties are generally indicated by walls, fences, hedges, ditches, etc., which occupy a considerable width upon the ground, being partly upon either side of the dividing line, upon which it therefore becomes impossible to apply the measuring standard. In such cases, it is usual to measure the opposite sides of imaginary parallelogram, equal offsets being made at the ends of the line, and

the measurements effected between the offset points. The offset angles are quite frequently estimated by the eye, and brought as nearly as possible to right angles. Even if these angles are instrumentally measured, the operation becomes complicated, thus increasing the liability to error. Impassable obstacles along boundary lines are of frequent occurrence, and various expedients, more or less complicated, are resorted to for ascertaining the distances through them. The difficulties of measuring accurately over uneven ground, requiring a careful and laborious use of the plumb line, the avoidance of sagging or unequal stretching when the chain is used, the exact marking on the ground of the end of the measuring standard, &c., are familiar to all land surveyors.

MEASUREMENTS OF ANGLES.—It is easy to see that the measurements of angles with good instruments can be performed with far greater ease and precision than the measurements of distances by the ordinary methods. Either mode of meas-

urement is merely a determination of *ratios*. Thus, when we measure the length of a line by direct applications of a standard length upon the ground, we simply ascertain the ratio between the length of the measured line and that of the chain. So, if we measure the three angles of a triangle, we know, by a simple computation, the ratio of its three sides to each other. The percentage of error in the ratios, as found by either method is much less in the measurement of angles with good instruments; for although the distances which are compared together in this measurement, namely, those marked in degrees along the limb of the instrument used, are many times smaller than those compared together in the measurement of distances upon the ground, the nicety of the mechanism and the ease of the verification by repetition admit a precision quite unattainable in actual ground measurements, except by slow and laborious processes.

TRIGONOMETRICAL SURVEY. — In the

trigonometrical survey this superiority of angle measurement is practically recognized. Convenient points are selected, at suitable distances from each other, where the angles between imaginary lines, joining adjacent points, can be measured. The whole area to be surveyed is cut up by these lines into a net-work of triangles, and the ratios of the sides of these triangles to each other are determined by measuring the angles between them. Then, when we ascertain the length of any one side of any one triangle, we can compute all the sides of every triangle, or the distances from point to point throughout the whole survey.

DEGREE OF ACCURACY ATTAINED.—The accuracy of the result, accordingly, depends upon the precision with which this one side or base line is measured, as well as upon the accuracy of the angle measurements. It is usual to verify the whole of the work by measuring another base line in a distant part of the net-work of triangles. For example, in the Coast Survey work, a base line was measured

at Fire Island, on the south side of Long Island, in 1834; another in Attleborough and Sharon, Massachusetts, in 1844; and another near the village of Epping, in Columbia, Washington county, Maine, in 1857. The distance between the Fire Island and Massachusetts bases, along the axis of triangulation, was 230 miles, and between the Massachusetts and Epping bases 295 miles. The length of the Fire Island base, 8,715,924 meters, or 5.415 miles, as actually measured, varies from the length, as computed from either of the other two bases, by less than 0.07 meters, or 2.75 inches; and the probable error of any computed line between these two bases is shown by careful analysis not to exceed $\frac{1}{288000}$ of its entire length. This proportion of error to distance amounts to 0.22 inches in a mile, or a little less than 2 feet in 100 miles.

This degree of accuracy indicates the wonderful skill which has been attained in the construction and use of instruments both for the measurement of base

lines and of angles. Thus, if we compare the actual distance upon the limb of a theodolite, 30 inches in diameter which corresponds to an error of $\frac{1}{288000}$, which we shall suppose to be all thrown into one of the three angles of a single well-conditioned or nearly equilateral triangle, we shall find it equivalent to about $\frac{1}{200000}$ of an inch,* being about 0.71 seconds of arc.

The percentage of error here developed is so small that it would not practically vitiate the measurement of lands even in the most valuable localities of great cities.

SUPERIORITY OF THE TRIGONOMETRICAL METHOD.—The degree of precision commonly attained in direct measurements of distances by ordinary methods falls very far short of this, and even of

* More exactly 0.000052 inches, for we have radius = 57.3°, nearly, in terms of arc; or making radius = unity, $1^\circ = \frac{1.0}{57.3}$, $1'' = \frac{1}{206200}$ and $\frac{1}{288000} = 0.71''$ nearly. Conversely in terms of distance, if radius = 15 inches, $\frac{15}{288000} = 0.000052$ inches.

that attainable in angle measurements with the ordinary surveyor's theodolite or transit.

It follows that accuracy, even in common surveying, would be promoted by using the method of triangulation for ascertaining ratios, whenever practicable or convenient, in preference to the common methods of traverse surveying. In the survey of a field or tract of land, for example, triangulation from one or two judiciously selected and carefully measured bases would give the positions of the corners and other objects with greater accuracy and far less labor than the usual routine of perambulating the outside boundaries, which is now taught in treatises on surveying, and generally practiced by surveyors.

ADVANTAGES OF COMBINING SURVEYING WITH GEODESY.—With the faculties afforded by the Coast Survey triangulations, when carried to the tertiary stage it is not difficult to perceive that a general method of determining the positions of points of by co-ordinates might be es-

tablished, and that while determinations thus made would be far more satisfactory and definite than those obtained by the ordinary methods of surveying, they would involve no more labor. Under such a system, the field work might consist of such a combination of triangulation and traverse surveying as would be found most convenient under the special circumstances.

To carry out such a system, the trigonometrical stations should be located so near together that two or more of them would be available for any subsequent local survey. If not otherwise visible there should be convenient arrangements for the temporary erection of signals upon stations to indicate their positions, making them visible from adjacent stations.

TRANSFORMATION OF CO-ORDINATES.—While for geographical purposes, the great co-ordinate planes already described are the most suitable for reference; simplicity and convenience would be promoted by transforming this general co-ordinate

system into numerous plane rectangular systems in limited local areas. Accordingly, instead of defining the position of a point by giving its terrestrial latitude and longitude, we would give its "latitude and departure," or its co-ordinates, from the zero point or origin of co-ordinates for the containing area.

For this purpose the local areas, into which the earth's surface is subdivided should be made sufficiently small to reduce the error which would arise from considering each separate area or plane surface, to an inconsiderable amount. Six miles square is not far from the average area of townships in the northern, middle, western, and most of the southern States. The bulge of curvature, or the versed sine of half the arc of six miles would be about six feet. This would make the proportional difference between the straight and the curved distance, or between the length of the cord and of the arc, much less than the percentage of probable error inseparable from measurements of the utmost attainable precision

in actual practice. Township boundaries, therefore, would seem to afford the most convenient divisions between separate co-ordinate systems. If, however, as is the case in a few of the Southern States, no smaller subdivisions than counties exist, these are not usually so large that the use of a single co-ordinate system over its areas would involve any important error.

DIRECTION OF AXES.—The most appropriate and convenient directions for the axes would doubtless be found in meridians and perpendiculars to them, since azimuths reckon from the meridional axes would then conform very nearly to the true astronomical azimuth. Owing to the convergence of meridians, there would be a small variation from the true azimuth, increasing with the distance from the axis, but no practical error need arise from this cause. The true azimuth, if needed, is easily computed by the formula,

$$\tan. \tfrac{1}{2} C = \sin. L \tan \tfrac{1}{2} P,$$

in which L is the middle latitude between

the origin and the point where the convergence is to be computed, P the difference of longitude between the same points, and C the convergence sought. In passing from one district to another however, a certain degree of complication arises, and it becomes necessary to take the convergence into account. We may, without practical error, consider any two adjacent districts as lying in a plane produced by developing a conical surface tangent to the earth on the middle parallel of latitude between the origin of the two districts. On this plane this convergence of the meridional axis of small districts will conform to the above formula, and the small angle of convergence measures the change of direction between the co-ordinate systems of the two districts.

PASSING FROM ONE DISTRICT TO ANOTHER.—In the passage from one district to another, four different cases of transformation of co-ordinates arise, namely:

1st. When the origin of the new system is east and north of the origin of the old system.

It will of course, be necessary, while assigning positive values to latitudes and departure in one direction, to give negative values in the opposite direction; thus if north and east are to be reckoned as positive, south and west must be reckoned as negative.

In this first case the old co-ordinates of the new origin are, accordingly, positive, while the new co-ordinates of the old origin are negative. Or, if we call the old co-ordinates a and b, and the new ones a' and b', a and b are here positive, and a' and b' negative.

2d. When the new origin is west and north of the old, or a negative, b positive, a' positive and b' negative.

3d. When the new origin is west and south of the old, or a and b negative, and a' and b' positive.

4th. When the new origin is east and south of the old, or a positive, b negative a' negative and b' positive.

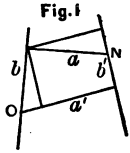
For ascertaining the new co-ordinates of the old origin, the equations,

$$-a' = a\cos.\psi + b\sin.\psi,$$
$$-b' = b\cos.\psi - a\sin.\psi,$$

would prove correct in all these cases, provided proper positive and negative values were given to the different terms in the equations. It is necessary to remember, however, that ψ, the angle of change in direction, between the old and new axis, is positive, according to trigonometrical usage, when reckoned from zero around towards the left, and negative in the opposite direction, while, according to the geodetic method of estimating azimuths, positive angles are reckoned around to the right.

To avoid the confusion which might arise from these different method of estimating angles, or from assigning a negative value to ψ, equations are given below for each of the four cases. Their correctness will be apparent on simple inspection of the accompanying figures, in which O and N are the old and new origins respectively:

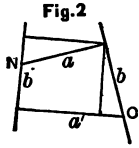
Fig. 1



CASE FIRST.

$$a' = a\cos.\psi + b\sin.\psi.$$
$$b' = b\cos.\psi - a\sin.\psi.$$

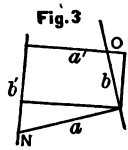
Fig. 2



CASE SECOND.

$$a' = a\cos.\psi + b\sin.\psi.$$
$$b' = b\cos.\psi - a\sin.\psi.$$

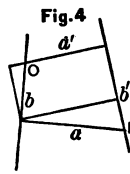
Fig. 3



CASE THIRD.

$$a' = a\cos.\psi - b\sin.\psi.$$
$$b' = b\cos.\psi + a\sin.\psi.$$

Fig. 4



CASE FOURTH.

$$a' = a\cos.\psi - b\sin.\psi.$$
$$b' = b\cos.\psi + a\sin.\psi.$$

In these equations the negative sign is omitted before a' and b', but we must remember that they are always estimated in directions opposite to those of a and b.

Since ψ is a very small angle, *cos.* ψ approximates closely to unity, and it appears by these equations that a' is greater than a, and b' is less than b when the new origin is farther *north* than the old, while a' is less than a, and b' greater than b when the new origin is farther *south* than the old.

Having found the new ordinates of the old origin, that is of O referred to N, the ordinates of any point referred to the new origin may be computed from its old ordinates by the equations:

$$x = x' \cos. \psi - y' \sin. \psi + a$$
$$y = x' \sin. \psi = y' \cos. \psi + b;$$

in which x = departure of any point as referred to N,

x' = departure of the same point as referred to O,

y = difference in latitude of any point as referred to N,

y' = difference in latitude of the same point as referred to O,

a = departure of the origin of O as referred to N,

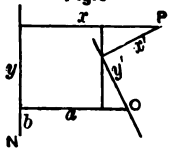
b = difference in latitude of the origin of O as referred to N,

ψ = the angle of convergence of the meridional axes of N and O.

These are the formulæ for passing from one system of rectangular co-ordinates to another in the same place.

In these equations, we may consider north and east to be the positive directions, as before, the opposite directions being negative. Also *sin.* ψ is positive when the change of axial direction is towards the left, and negative when in the opposite direction; that is, positive when N is farther east than O, and negative when farther west. *Cos.* ψ will always be positive, since ψ is always either in the first or fourth quadrants.

Fig. 5



In Fig. 5, the point P, of which x and y are the new co-ordinate, is east and north of the axis passing through O, making x and y greater than a and b respectively, and all the terms

of both equations have, accordingly, positive values. But if x, x' or a be estimated towards the west, its sign must be reversed when particular values are substituted in the equations; likewise, if y, y' or b have a southern direction, its sign must be reversed.

CO-ORDINATES IN A SINGLE DISTRICT.—It is hardly necessary to say that when the azimuth and distance are given from a point whose co-ordinates are known to any other points, the co-ordinates of the latter are found by multiplying the given distance by the sine and cosine of the azimuth, the first product giving the departure and the other the difference of latitude.

VERIFICATIONS.—Before finally establishing the co-ordinates of a survey its accuracy should be tested in the most rigid manner, both as regards the instrumental observations and the computations. The computations are easily verified by working to the same point from different directions. Some methods of verifying the field work are indicated

in the accompanying plates and explanations. Others will suggest themselves to the surveyor under different circumstances.

ILLUSTRATIONS OF NOTATION.—Plates I. and II. exhibit the sort of notation which may be employed under the system proposed. Instead of magnetic bearings or angles written between lines, azimuths are given, which are estimated around to the right from zero at due north to 360° or due north again. From these azimuths angles around to the right are easily found by subtracting the first azimuth from the second, adding 360° to the latter if zero comes between.

The co-ordinates are here given in feet, but metres, chains or any other standard units may be used in the same way. The letters N. E. S. W. indicate the directions for the origin, north, east, south or west. Points upon the division lines between two adjacent districts have their co-ordinates given in both. For railroad surveys, it would be found convenient in plotting to have the co-ordinates of

tangent points and centers of curves given, even though the latter should not appear upon the plan. Other convenient details of notation will suggest themselves to engineers, and indeed the plates are only intended to present to the eye the general features of the method proposed. A great variety of cases will arise in practice, many of them requiring special treatment.

COMPUTATION OF AREAS.—Areas are computed under this system with special facility and certainty, the method being the common one of double latitudes and departures. This method is prescribed by law in the State of Ohio, for calculating areas of farming lands and for testing the accuracy of surveys made with the surveyor's compass.

DESCRIPTIONS FOR CONVEYANCES.—For definite and accurate descriptions of land in conveyances, it does not seem possible to devise a more precise and certain method than that of co-ordinates from geodetically determined reference points of origin. The present loose and indefi-

nite descriptions in conveyances, upon which the tenure of a large part of the real estate of the county now depends, are disgracefully uncertain, and frequently lead to excessive expenses of unnecessary litigation, and sometimes to costly errors of misplaced constructions.

CONVENIENCE IN CONSTRUCTING MAPS.—In the construction of maps and plans, co-ordinate determinations will be found especially convenient. After completing the survey of any portion of a district, it is easy to place it in its proper position upon the map of the entire district, with the certainty that other portions subsequently surveyed will fit into their proper places, without the perplexity and the distortions frequently accompanying the attemps to unite two or more independent surveys made under the methods in common use.

SUMMARY OF ADVANTAGES.—A few of the advantages which may be expected to follow the general adoption of the co-ordinate method of surveying may be summed up as follows:

First.—The attainment of the highest practicable degree of accuracy as well in smaller local surveys as in more extended operations. The units of measurement which form the basis of the United States Coast Survey have been most carefully compared with those of the entire civilized world, and with the dimensions of the earth itself, and are verified to a degree of precision beyond which the present attainments of scientific skill have not passed.

Second.—Extreme simplicity of notation, with ease and convenience of field work and computation.

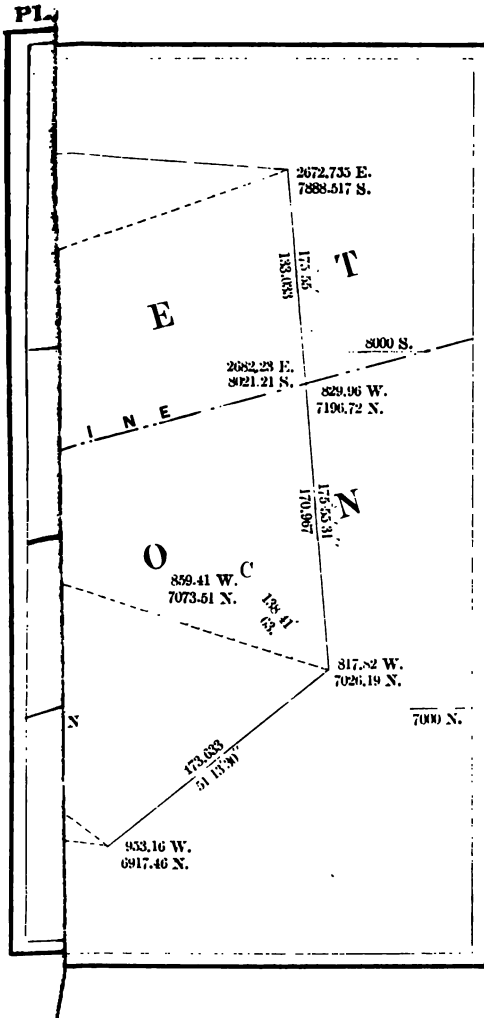
Third.—Facility in graphic represention.

Fourth.—Absolute certainty of locations in descriptions for conveyances, and consequent removal of a fruitful cause of litigation and trouble.

ALTITUDES.—No change is proposed in the existing methods of determining the third ordinate or altitude. The most convenient mode of fixing this ordinate pre-supposes that the form of the earth's surfaces, or of that surface which would

be presented if the irregular surface of the land were raised to the level of the sea, has been accurately determined by geodetic operations, so that in ordinary surveying we have only to ascertain the heights of our points of survey above this imaginary level. This is done by using the spirit level, by measuring vertical angles, and by barometrical observations. The first method admits the greatest degree of precision under ordinary circumstances, and is almost exclusively used for engineer purposes.

Plate I. represents a tract of land lying partly in Pomfret and partly in Weston, two adjacent towns. One side is bounded by a lake, and a road passes through the tract. Several of its corners are visible from the point A. The point B, where the town line crosses the west side of the road, is one of the stations fixed by the preliminary trigonometrical survey. All the co-ordinates upon this plan have been determined by computing the latitudes and departures, directly or indirectly from this point. The con-


P1.
2672.735 E.
7888.517 S.
133.033
175.55
E
T
8000 S.
2682.23 E.
8021.21 S.
829.96 W.
7196.72 N.
I N E
175.53.31"
170.967
N
O
C
859.41 W.
7073.51 N.
134.41
63.
817.82 W.
7026.19 N.
7000 N.
N
473.833
51 15'30"
933.16 W.
6917.46 N.

vergence of the axial meridians in the two towns is 31″, and the co-ordinates of the origin in Weston referred to Pomfret, are,

$$a = 3513.27 \text{ E, and } b = 15217.81 \text{ S.}$$

From these we can compute the ordinates of the Pomfret origin referred to Weston, by the formulæ for Case Fourth:

$$a' = a \cos. \psi - b \sin. \psi,$$

$$b' = b \cos. \psi + a \sin. \psi,$$

substituting values

(cos. $31'' = 1$; ψ log. sin. $31'' = 6.1769365$)

$a' = 3513.27 - 15217.81 \sin. 31'' = 3510.983$

$b' = 15217.81 + 3513.27 \sin. 31'' = 15218.338$

Reversing the directions,

$$a = 3510.983 \text{ W. } b = 15218.338 \text{ N.}$$

Equations for passing from Pomfret into Western.—The general formulæ are:

$$x = x' \cos. \psi - y' \sin. \psi + a.$$

$$y = x' \sin. \psi + y \cos. \psi + b.$$

In this case $\psi = +31''$ (in passing eastward from Pomfret to Weston the axis swing around to the *left*, and the angle of change is *positive*, according to trigo-

nometrical usuage): $a = 3510.983$, and $b = 15218.328$. Substituting values, the equations become:

$$x = x' - y \text{ sin. } 31'' - 3510.983$$

$$y = x' \text{ sin. } 31'' + y + 15218.338.$$

Equations for passing from Weston to Pomfret.—In this case $\psi = -31''$ (reckoned to the right), $a = 3513.27$, and $b = 15217.81$; substituting values:

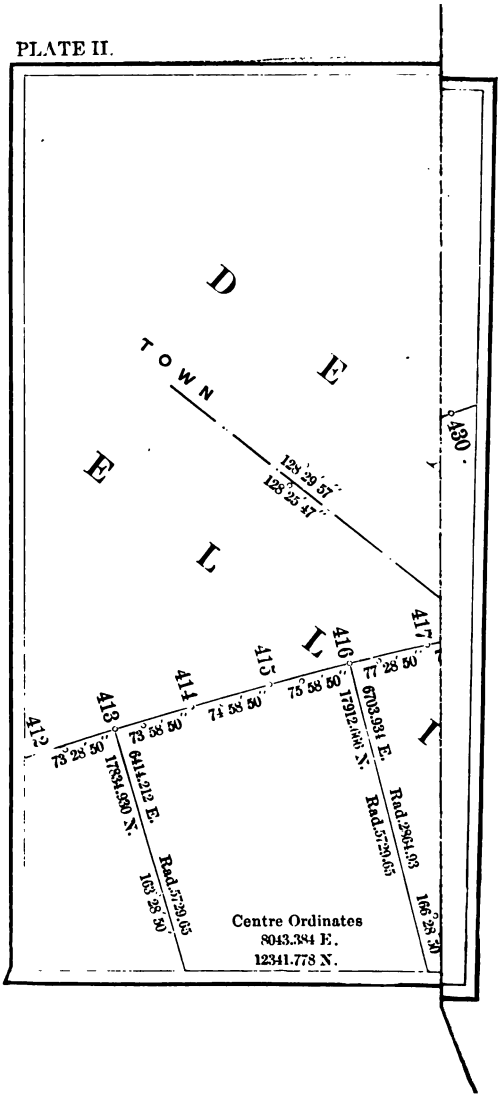
$$x = x' + y' \text{ sin. } 31'' + 3513.27$$

$$y = -x' \text{ sin. } 31'' + y' - 15217.81.$$

Double pairs of co-ordinates are given along the town line, and either pair may be computed from the other by using these equations. The accuracy, both of the equations and the computations are verified by reversing the method.

Verification of the Survey.—At the point C, where three trigonometrical stations can be seen, azimuths were taken to each and the co-ordinates of C computed by the "three point problem." They are 859.36 W, and 7073.59 N, from the origin of Weston. The Azimuth from C to the southeast corner of the tract is 138°41′,

PLATE II.



D
E
E
L
L
I
TOWN
128° 29′ 57″
128° 25′ 47″
430
417
416
415
414
413
412
77° 28′ 50″
75° 58′ 50″
74° 58′ 50″
73° 58′ 50″
73° 28′ 50″
6703.934 E.
17912.666 N.
Rad. 2864.93
Rad. 5729.65
166° 28′ 50″
6414.212 E.
17834.930 N.
Rad. 5729.65
163° 28′ 50″
Centre Ordinates
8043.384 E.
12341.778 N.

and the distance 63 feet, giving the coordinates of C, 859.41 W, 7073.51 N. The degree of accuracy here indicated would probably be sufficient under ordinary circumstances.

Plate II.—In running railway surveys, every opportunity should be taken to connect with the trigonometrical stations which become accessible near the line, so as to verify its direction and the position of the stakes or stations. The methods of doing this by triangulation and otherwise are simple, and will readily suggest themselves to the engineer. This plate illustrates the passage of a railroad survey across a town line, passing from one system of ordinates to another.

The ordinates of the origin in Dexter referred to Elliot, are:

28243.13 E. 31497.21 N.

and the convergence is 4′ 10″.

By the equations given for Case First we find the ordinates of Elliot referred to Dexter to be

(cos. 4′ 10″=1, log. sin. 4′ 10″
=7.0819376.)

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